More than 1-D. Conservation laws in 2-D or 3-D.
Flux q is now a vector. Use Gauss theorem to obtain general form \( \rho_t + \text{div}(q) = S \) (S equal sources & sinks).

Examples of S:
- Cars flowing in/out of highway through commuter township.
- Water flowing into river from small affluents.
- Heating by electromagnetic waves [microwave oven].

EXAMPLE: Heat flow in 2-D or 3-D.

Then, 
\[
\rho = r \ c_v \ T = \text{conserved stuff (heat) per unit mass}
\]

Where:
- \( c_v \) = specific heat of material
- \( r \) = mass density
- \( T \) = temperature

Fick’s law applies:
\[
Q = - \ \kappa \ \text{gradient}(T), \text{and } \kappa = \text{heat conductivity.}
\]

Thus Heat equation:
\[
T_t = \nu \ \text{Laplacian } T,
\]

Where
\[
\nu = \kappa / (r \ c_v) = \text{coeff. thermal diffusion.}
\]

EXAMPLE:
Diffusion equation (Salt in water, sugar in coffee, ink in water, ETC.)
Same as heat equation
\[
C_t = \nu \text{*Laplacian } C
\]

Where
- \( C \) = concentration (salt, ink, sugar, ...)
- \( \nu \) = diffusion coefficient.

Intuitive justification of Fick’s law.

DIMENSIONS and DIMENSIONAL ANALYSIS.
What are the dimensions of kappa? nu? \( c_v \)?

How long does it take sugar to sweeten a coffee cup without stirring? Idealized problem: start with a very small blob of ink, and ask:

What is the radius of the blob of ink, \( R = R(t) \), as the blob expands due to diffusion?

Dimensional analysis says: \( R(t) \propto \sqrt{\nu t} \)

In particular, let \( L \) be the size of the coffee cup. Sugar will reach whole cup when \( R = O(L) \implies \text{time} = O(L^2/\nu) \).

Also the relevant time needed to cool/heat a size \( L \) vessel.
These times are very long when measured in human-relevant scales. Hence stirring needed. Boiling and convection speed up heating.

Questions:
- why does stirring help?
- why does convection occur?
- what would happen when heating something with a flame in the absence of gravity?

At room temperature, in cm$^2$/sec

Thermal diffusivity: water $\sim 0.0014$ mercury $\sim 0.042$
Diffusion in water: NaCl $\sim 10^{-5}$