HW1 posted online (late last night), due 9/21 (W, since "student holiday" on M)

Thm (van der Waerden)

\[ \forall K, l, \exists n = n(K, l) \text{ s.t. every } K\text{-coloring of } \mathbb{N} \text{ contains a length } l \text{ monochromatic arithmetic progression} \]

"we didn't go into bounds, but if you look, it just becomes towers of 2s all over the place."

Thm \[ n(2, l) \geq 2^{l/2} \text{ (pf from Jukna p.230)} \]

Pf: Color \[ [n] \text{ randomly. } \Pr(\text{given } l\text{-term seq. is mono}) = \frac{1}{2^{l-1}} \]

\# such progressions < \[ \binom{n}{l} \] \text{ (1st+2nd in seq.)}

So prob(mono) < \[ \frac{1}{2^{l}} \binom{n}{l} \], so ETST \[ 2^{-l} \binom{n}{l} < 1 \]

\[ n = 2^{l/2} \Rightarrow \checkmark \]

Thm (Erdős - Szekeres, 1935) \[ \forall K \exists n = n(k) \]

\[ \forall n\text{-point sets in } \mathbb{R}^2 \text{ in general position, } \exists \text{ a } k \text{subset in convex position.} \]

general position: no 3 in same line

\[ n(3) = 4 \quad n(4) = 9 \quad (?) \quad \ldots \]

Pf: \[ n(k) = \mathbb{R}^2(3, k) \text{ (edges = 3-subsets, 2-colors, want } \geq k \)

First proof: color triangles by orientation \[ \triangle \]

i < j < k \Rightarrow one color, i < k < j \Rightarrow \text{ other. Then mono subset is convex, since } \varnothing \] \text{ one of inner triangles contradicts clockwise- or counterclockwise-ness}
Second pf (Jukna p322): Let \( \alpha(a,b,c) = \# \text{ of } b \) interior pt. of \( a \triangle b \)
\[ \delta(a,b,c) = \begin{cases} 1 & \alpha(a,b,c) > 0 \\ 0 & \text{o/w} \end{cases} \]
\( \alpha(abc) = \alpha(abd) + \alpha(acd) + 1 \)
(in fact, this also tells you all subtriangles have same parity)

Defn \( \mathcal{F} = \{A_1, A_2, \ldots \} \) \( A_i \subset [n] \)
\( \mathcal{F} \) is \( K \)-colorable if \( \exists X : [n] \rightarrow \mathbb{K} \) s.t.
\( \forall i \ A_i \) is not monochromatic

Thm If \( |A_i \cap A_j| = 1 \) \( \forall i, j, A_i, A_j \in \mathcal{F} \)
then \( \mathcal{F} \) is 2-colorable

Defn \( \mathcal{F} \) is \( K \)-uniform if \( \forall i \ |A_i| = K \)

Pf: (greedy) (\( \checkmark \)) (by o/w \( \exists i \) s.t.
\( i \in A_p \), \( i \in A_q \), \( A_p \)
all red but i, \( A_q \) all blue, etc.)