\[ \Gamma_{n,k} = \text{graph of } k\text{-colorings of } G_{n,n}. \]

Then what's the diameter of \( \Gamma_{n,k} \) ?

(remember height function argument)

\[ \text{if } l = \sum_{i,j} f(i,j) = O(n^2) \text{ (since each entry } O(n) \text{)} \]

\[ \text{then each decrease is size } 2 \Rightarrow \]

\[ \text{diam}(\Gamma_{n,k}) = O(n^3) \]

OTOH, \( \text{diam}(\Gamma_{n,k}) = \Theta(n^2) \) for \( k \approx \frac{n}{2} \)

and we see \( \text{diam}(\Gamma_{n,k}) = O(n^3) = \text{diam}(\Gamma_{n,n}) \)

Theorem

\[ G_{n,n} \text{ graph w/ } m \text{ edges } \Rightarrow \]

\[ \chi(G) \leq \frac{n}{2} + \sqrt{2m + \frac{n}{4}} \]

pf: \[ m \geq \binom{n}{2} \approx \frac{n^2}{2} \text{ since in minil proper coloring } \]

1 edge between any two colors

One More Obvious Theorem

\[ \chi(G) \geq \sqrt[3]{\chi(G)} \]

pf: Duh.