Thm: G finite group, \( G = \langle \alpha, \beta, \gamma \rangle \), \( \alpha^2 = \beta^3 = \gamma^3 = 1 \)

\[ \Gamma = \Gamma(G, \{\alpha, \beta, \gamma\}) \text{ Cayley graph} \]

contains h.c.

Defn of Cayley graph

\( G \) finite q.p.

\( S \subseteq G \)

\[ \Gamma(G, S) \text{ is Cayley graph when} \]

\[ V(\Gamma) = \{ g \in G \} \quad E(\Gamma) = \{ gS \mid g \in S \} \]

(Clearly \( \langle S \rangle = G \iff \Gamma \) is connected (yup, it's clear)

Pf of Thm: \( H = \langle \beta, \gamma \rangle \) has a Cayley

graph big cycle w/ alternating \( \beta \) \( \gamma \) edges

Look at cosets of \( \Gamma(H) \) w/ in \( \Gamma(G) \).

Note: Note they're 2-regular, each \( \alpha \) in \( \Gamma(G) \)

is 3-regular

\[ \text{can hook up cycles like this, if not h.c.} \Rightarrow \]

\[ \exists \alpha \text{ edge going out} \Rightarrow \]

hook up again \( \Rightarrow \sqrt{\text{ }} \]

Thm (Menger) (in MGT p75) \( \text{not} = \# \text{out} \)

1) \( g \neq t \in V(G) \Rightarrow \min \# \text{ vertices separating } s + t \)

\[ = \max \# \text{ independent } s + t \text{ paths (vertex disjoint) except }s + t \]

2) \( \min \# \text{ edges separating } s \text{ from } t = \max \# \text{ edge disjoint } s + t \text{ paths } \]

\[ A, B \subseteq V \quad \min \# \text{ edges separating } A \text{ from } B \]

\[ = \max \# \text{ vertex-disjoint paths from } A \text{ to } B \]
$3 \rightarrow 1$ (Kind of) if $A = \{c\}$, $B = \{e_3\}$ then

\[ \min = 1 \quad \text{(sort)} \]

Better: let $A = N(c)$ and $B = N(e_3)$

$3 \Rightarrow 2$ Take $L(G)$, $V(L(G)) = E(G)$

\[ E(L(G)) = \{ (e_1, e_2) | e_1 \cap e_2 = \emptyset \} \]

Thm $|G| \geq 3$, $K(G) = \alpha(G)$, $(K(G)) = \nu_x$-connected.

$\Rightarrow$ $G$ contains b.c.

Pf Let $C$ be the longest cycle, $v \in V - C$

\[ P = \{ p_1, p_2, \ldots, p_m \} \text{ vertex-disjoint } \nu - c \text{ paths} \]

\[ |P| = l > \min \{ l|C| \} \quad \text{by II's Thm} \]

Also $l < |C| \Rightarrow l \geq K$, Can't have

or $C$ or $C$

$\exists$ at least $K$ pts next to pts connected to $v \Rightarrow$ they $\cup v$ form

indep set of size $K + 1$. $\square$

(This is Thm 10.1.2 in Diester's G0)

(link on webpage)