Pattern Avoidance

$\omega \in S_k$

Def: $\omega \in S_n$ contains $\alpha$ if there exists $i_1, \ldots, i_k$ such that $\alpha(i_p) < \alpha(i_q)$ iff $\omega(p) < \omega(q)$

$\omega$ avoids $\alpha$ otherwise

Thm

$\forall \omega \in S_3 \mid \omega$-avoiding permutations $\sigma \in S_n$ is

$C_n = \frac{1}{n!} \binom{2n}{n}$

Pf: Note that, by subtracting from not or reversing order, $\exists$ 231-avoiding $= \exists$ 312-avoiding and $\exists$ 321-avoiding $= \exists$ 123-avoiding, etc.

So just consider $\exists$ 123+ (23).

312-avoiding: 6 $\in \{312\}$ avoiding $\Rightarrow \sigma(i)$ is in the middle somewhere, everything to the right must be smaller than everything to the right.

Call w have 312 $\in \pi$.

Build binary tree w/ root 1, left child is min on left, right child is min on right, etc.

# binary trees = $C_n$.

321 avoiding: Dilworth $\Rightarrow$ can be decomposed into two increasing subsequences. Then RSK comes to build pair of tableaux, convert these into lattice paths to get a bijection between 321-avoiding + Dyck paths.

So $\omega$-avoiding is $e^{\omega}$ (Stanley-Wilf conjecture). (Proof next time)