4 Make it your business to track down to high accuracy, in two ways starting from the initial guess $z_0 = 1 + i$, that root of

$$z^4 + z + 1 = 0$$

which resides in the first quadrant of the complex z-plane. Employ

(a) the complex Newton method, and less efficiently also

(b) some real variable search for that $x,y$ pair which solves simultaneously the related pair of equations

$$x^4 - 6x^2y^2 + y^4 + x + 1 = 0, \quad 4x^3y - 4xy^3 + y = 0.$$ 

5 The well-known Legendre polynomials

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = (3x^2 - 1)/2, \quad P_3(x) = (5x^3 - 3x)/2, \ldots$$

obey the recurrence relation

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x).$$

Hence locate via Newton's method to at least 6 decimals that root of $P_{30}(x)$ which lies closest to $x = 0.5$, left or right.

6 As implied by the diagram overleaf, the famous quadratic iteration

$$x_{n+1} = Cx_n(1-x_n) = g(x_n)$$

settles down to a stable two-hop cycle when the constant $C$ exceeds 3 but does not exceed an upper critical value roughly equal to 3.45.

Your task: Analyze the stability of the "stroboscopic" iteration

$$x_{n+2} = g[g(x_n)] = G(x_n),$$

and thereby locate the precise value of $C$ at which this related two-hop cycle bifurcates in turn.
Results of the iteration

\[ x_{n+1} = C x_n (1 - x_n) \]