16. Calculate the **roof area** of Kresge auditorium, working from the theory that this familiar object is shaped like a paraboloid of revolution

\[ z = \frac{a^2 - x^2 - y^2}{2a}, \]

truncated by vertical cylinders of radius \( \sqrt{3}a \) centered on the opposite vertices.

17. In the spirit of **Gaussian quadrature**:

(a) determine **polynomials** \( p_0(x), p_1(x) \) and \( p_2(x) \) such that

\[ \int_{0}^{1} \sqrt{x} p_m(x) p_n(x) \, dx = 0 \quad \text{for} \quad m \neq n, \]

(b) find **weights** \( w_1 \) and \( w_2 \) such that the estimate

\[ \int_{0}^{1} \sqrt{x} f(x) \, dx = w_1 f(x_1) + w_2 f(x_2) \]

based on the roots \( x_1 \) and \( x_2 \) of \( p_2(x) \) becomes exact for all **cubic** polynomials, and

(c) finally **test** this fancy folderol on the integral \( \int_{0}^{1} \frac{1}{\sin x} \, dx \).

18. Evaluate the sum

\[ S = \sum_{k=1}^{\infty} \left(\frac{1}{x_k}\right)^2, \]

where \( x_k \) is the \( k \)-th positive root of \( x = \tan x \).

Work carefully here, and employ sensible extrapolations or some other finesse like

\[ 1 + 1/9 + 1/25 + 1/49 + 1/81 + \ldots = \pi^2/8. \]

Then you should find that this sum \( S \) equals a very simple fraction!