18.335 Problem Set 2

Problem 1: Stability
(a) Trefethen, exercise 15.1. [In parts (e) and (f), assume that $\frac{1}{q}$ can be computed to $O(\varepsilon_{\text{machine}})$ and concentrate on the accumulation of errors in the summations.]
(b) Trefethen, exercise 16.1.

Problem 2: Norms
(a) Derive Trefethen eq. (3.10) (for which Trefethen only writes “by much the same argument”). Find the code that computes the induced $\|A\|_\infty$ norm in Julia, the `opnorm(A, Inf)` function, on github.com/JuliaLang/julia in stdlib/LinearAlgebra/src/generic.jl and satisfy yourself that it is equivalent to (3.10).
(b) Trefethen, problem 3.4. Check your result for a random $10 \times 7$ matrix $A$ in Julia, constructed by $A=\text{randn}(10,7)$ with the induced $p=2$ norm as computed by `opnorm(A)` in Julia.

Problem 3: Conditioning
In class, we found the condition number of $f(x) = Ax$, in the $L_2$ norm, to be $\|A\|_2\|x\|_2/\|Ax\|_2$ for some $A \in \mathbb{C}^{m \times n}$ and $x \in \mathbb{C}^n$. Instead, compute the condition number of $g(A) = Ax$, using any convenient choice of norm (e.g. the Frobenius norm for the input matrices $A$).

Problem 4: SVD and low-rank approximations
(a) Trefethen, problem 4.5.
(b) Trefethen, problem 5.2.
(c) Trefethen, problem 5.4.