1. 12.5.1
Solution: See R script/html Problem_12_5_1.r/html

2. 12.5.6
Prove this version of the Bonferroni inequality:
\[ P(\bigcap_{i=1}^{n} A_i) \geq 1 - \sum_{i=1}^{n} P(A_i^c) \]
Let \( A_* = \bigcap_{i=1}^{n} A_i \). Then
\[ A_*^c = \bigcup_{k=1}^{n} A_i^c \]
So \( P(A_*^c) = P(\bigcup_{k=1}^{n} A_i^c) \leq \sum_{k=1}^{n} P(A_i^c) \)
Because \( P(A_*) = 1 - P(A_*^c) \), it follows that
\[ P(A_*) \geq 1 - \sum_{k=1}^{n} P(A_i^c) \]
In the context of simultaneous confidence intervals
\( A_i \) is the event that the \( i \)th confidence interval contains the associated parameter
\( \bigcap_{i=1}^{n} A_i \) is the event that all confidence intervals contain the associated parameters simultaneously.

3. 12.5.17. Find the mle’s of the parameters \( \alpha_i, \beta_j, \delta_{ij}, \) and \( \mu \) of the model for the two-way layout.
The model for the two-way layout is given by:
\[ Y_{ijk} = \mu + \alpha_i + \beta_j + \delta_{ij} + \epsilon_{ijk} \]
where
\[ \sum_{i=1}^{I} \alpha_i = 0 \]
\[ \sum_{j=1}^{J} \beta_j = 0 \]
\[ \sum_{i=1}^{I} \delta_{ij} = \sum_{j=1}^{J} \delta_{ij} = 0 \]
As noted in Rice (p. 493) the log likelihood is
\[ l = -\frac{LJK}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K (Y_{ijk} - \mu - \alpha_i - \beta_j - \delta_{ij})^2 \]

Solving the following equations:
\[
\frac{\partial l}{\partial \mu} = 0 \implies \hat{\mu} = \overline{Y}.
\]
\[
\frac{\partial l}{\partial \alpha_i} = 0 \implies \hat{\alpha}_i = \overline{Y_i} - \hat{\mu}.
\]
\[
\frac{\partial l}{\partial \beta_j} = 0 \implies \hat{\beta}_j = \overline{Y_j} - \hat{\mu}.
\]
\[
\frac{\partial l}{\partial \delta_{ij}} = 0 \implies \hat{\delta}_{ij} = \overline{Y_{ij}} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j.
\]

These terms yield the answer formulas of the problem.


5. 13.8.25. See R script html Problem_13.8.25.html