1. Let $P$ put probability $1/3$ each at the vertices of an equilateral triangle in the plane. Show that the spatial median of $P$ is at the center (centroid) of the triangle. *Hint:* by Euclidean transformations we can assume that the triangle has vertices $V_1 = (-1,0)$, $V_2 = (1,0)$, and $V_3 = (0,\sqrt{3})$, so that the centroid is at $(0,1/\sqrt{3})$. If a point $(x,y)$ is a spatial median, show by symmetry that $(-x,y)$ is also one, and then by convexity that $(0,y)$ is, in fact by strict convexity $x$ must equal 0, so the spatial median is on one of the perpendicular bisectors of the triangle.

2. Let $P$ put mass $1/3$ at each vertex $V_i$ of an obtuse triangle, where the angle at the obtuse vertex $V_2$ is more than $120^\circ$. Then show that the spatial median equals $V_2$. *Hint:* By the proof in the handout it’s enough to show that it gives a local minimum.

3. A function $f$ defined for samples $Z_1, ..., Z_n$ of points in the plane $\mathbb{R}^2$ will be called an *affinely equivariant location estimator* if for any non-singular transformation ($2 \times 2$ matrix) $A$ and $v \in \mathbb{R}^2$ we have $f(AZ_1 + v, ..., AZ_n + v) = Af(Z_1, ..., Z_n) + v$. Show that the spatial median is not affinely equivariant. *Hint:* Consider the examples in problems 1 and 2, and triangles with vertices at $(-1,0)$, $(0,y)$, and $(1,0)$ as $y$ varies.

4. Let $P = N(\mu, \sigma^2)$ and $Q = N(\nu, \sigma^2)$ be two normal distributions on the line with the same variance. Evaluate the Kullback-Leibler distance $I(P,Q)$ (defined just before Theorem 3.3.15) as a function of $|\mu - \nu|$.

5. Problem 1 of the “M-estimators and their consistency” handout.