Transformation doesn't map straight lines to straight lines.

\[ y^2 = x^3 + ax^2 + bx + c = f(x) \]

if coefficients \(a, b, c\) are rational, then \(f(x)\) has at least 1 real root.

\[ f(x) = (x-2)(x^2 + bx + c) \]

\(x \neq 2\) real root of \(f\).

These pictures are valid only if the roots of \(f\) are distinct.

Def \( F(x, y) = y^2 - (x^3 + ax^2 + bx + c) = 0 \) is nonsingular if

\[
\frac{\partial F}{\partial x}, \frac{\partial^2 F}{\partial x^2} \text{ are never both equal to 0 for } (x_0, y_0).
\]
Let \((x_0, y_0)\) be a singular point on \(F\):
\[
\frac{dF}{dy} \bigg|_{y_0} = 0 = 2y \quad \frac{dF}{dx} \bigg|_{x_0} = 0 = f'(x)
\]
\[y = 0\]
\[y_0^2 = f(x_0) = 0 \quad x_0 \text{ is a root of } f'(x_0) = 0\]

If \(f\) has a double root, then \(y^2 = x^2(x + 1)\)

\[y = x\]
\[y^2 = x^3\]

\[y = r^2 - r\]
\[r = \frac{y}{x}\]
\[r^2 = x + 1\]
\[x = r^2 - 1\]
\begin{align*}
y &= \frac{t}{2} x,
\eta &= \frac{t}{2} x_0,
\eta^2 &= x_0^2,
\frac{t^2}{\alpha^2} x_0^2 &= x_0^2
\end{align*}
\begin{align*}
x &= \frac{\alpha}{t} y,
y_0^2 &= \frac{\alpha^3}{t^2} y_0,
y_0 &= \frac{t^3}{\alpha^2}
\end{align*}