1. 18.712 TAKEHOME ASSIGNMENT

1. Let $Q$ be a quiver, i.e. a finite oriented graph. Let $A(Q)$ be the path algebra of $Q$ over a field $k$, i.e. the algebra whose basis is formed by paths in $Q$ (compatible with orientations, and including paths of length 0 from a vertex to itself), and multiplication is concatenation of paths (if the paths cannot be concatenated, the product is zero).

   (i) Represent the algebra of upper triangular matrices as $A(Q)$.
   (ii) Show that $A(Q)$ is finite dimensional iff $Q$ is acyclic, i.e. has no oriented cycles.
   (iii) For any acyclic $Q$, decompose $A(Q)$ (as a left module) in a direct sum of indecomposable modules.
   (iv) Find a condition on $Q$ under which $A(Q)$ is isomorphic to $A(Q)^{op}$, the algebra $A(Q)$ with opposite multiplication. Use this to give an example of an algebra $A$ that is not isomorphic to $A^{op}$.

2. Classify irreducible representations of the group $GL_2(\mathbb{F}_q) \times \mathbb{F}_q^2$ of affine transformations of the 2-dimensional space over a finite field, and find their characters.

3. Compute the decomposition into irreducible representations of all the induced representations from the cyclic subgroups of the preimage $\Gamma$ of $A_5 \subset SO(3)$ (the group corresponding to the affine Dynkin diagram $\tilde{E}_8$).

4. Find the multiplicities of the irreducible representations of $sl(2)$ in $V^\otimes n$, where $V$ is the 2-dimensional vector representation.