18.905 Problem Set 6
Due Wednesday, October 18 in class

1. Use the universal coefficient theorem to compute $H_\ast(L(p, q); \mathbb{Z}/m)$ for all $m$, where $L(p, q)$ are the lens spaces defined in class.


3. Given an arbitrary finitely generated abelian group $M$, compute $\text{Tor}_k(\mathbb{Q}/\mathbb{Z}, M)$ for all $k \geq 0$. (Bonus marks for doing an arbitrary abelian group.)

4. The $n$-dimensional chains $C_n$ form a functor from spaces to abelian groups. Suppose $F$ is another functor from spaces to abelian groups. Show that any natural transformation $\Theta$ from $C_n$ to $F$ must be as follows: For any space $X$, the map $\Theta_X : C_n(X) \to F(X)$ is given by

$$\Theta_X \left( \sum m_\sigma \sigma \right) = \sum m_\sigma F(\sigma)(\Theta_{\Delta^n} \Delta^n).$$

(Here we recall that the chain $\Delta^n \in C_n(\Delta^n)$ is represented by the identity map from $\Delta^n$ to itself.) Conversely, given an element $S \in F(\Delta^n)$, show that we can define a natural transformation $\Theta$ from $C_n$ to $F$ by

$$\Theta_X \left( \sum m_\sigma \sigma \right) = \sum m_\sigma F(\sigma)(S).$$