Due to President’s day, I believe this Monday’s class is rescheduled to Tuesday.

1. Verify the following isomorphisms:
   (a) For $X \in \text{Top}_*$, show that there is an isomorphism
       \[ \pi_{n-1}(\Omega X) \cong \pi_n(X). \]
   (b) Let $X$ be an unbased space. The unreduced suspension $\text{Susp}(X)$ is the space obtained from $X \times I$ by identifying all of the points in $X \times \{0\}$ and all of the points in $X \times \{1\}$. We do not identify points in $X \times \{0\}$ with points in $X \times \{1\}$. Show that there is an isomorphism
       \[ H_{n+1}(\text{Susp}X) \cong H_n(X) \]
       for $n$ greater than or equal to 1.

2. Hopf fibration. The purpose of this problem is to verify that there exists a non-trivial element of $\pi_3(S^2)$. The Hopf fibration is a map $\eta : S^3 \to S^2$. It is defined by viewing $S^2$ as $\mathbb{C}P^1$, and $S^3$ as the unit sphere in $\mathbb{C}^2$. The map $\eta$ is then defined by
       \[ \eta(x, y) = [x : y]. \]
       (Here, $[x : y]$ denotes the complex line in $\mathbb{C}^2$ spanned by the vector $(x, y)$.)
   (a) Let $X$ be the CW complex given by attaching a 4-disk along $\eta$.
       \[
       \begin{array}{c}
       S^3 \\
       \eta \\
       \Downarrow \\
       \mathbb{C}P^1 \\
       \Rightarrow \\
       D^4 \\
       \Rightarrow \\
       X.
       \end{array}
       \]
       Show that $X$ is homeomorphic to $\mathbb{C}P^2$.
   (b) Show that if $\eta$ is null homotopic, then $X$ is homotopy equivalent to $S^2 \vee S^4$.
   (c) Deduce that $\eta$ cannot be null homotopic by computing the cup product structure on $H^*(X)$.

3. Problem 3 on p358 of Hatcher.
4. Problem 1 on p79 of May.