18.950 Homework 5

1. (10 points) Let $f$ be a hypersurface patch. Suppose that $f$ lies in the half-plane $\{y_{n+1} \geq 0\} \subset \mathbb{R}^{n+1}$, and that $f$ is tangent to the hyperplane $\{y_{n+1} = 0\}$ at $x = 0$. Prove that then, the principal curvatures at $x = 0$ satisfy $\lambda_i \lambda_j \geq 0$ for all $i, j$.

2. (3 points) Let $f$ be a hypersurface patch of the form $f(x) = (x, \phi(x))$ for some $\phi : U \to \mathbb{R}$. Suppose that at the origin $x = 0$, both $\phi$ and $D\phi$ vanish. Compute the Christoffel symbols and their (first order) derivatives at that point.

3. (7 points) Let $f : U \to \mathbb{R}^3$ be a surface patch. Define the parallel surface at distance $\epsilon$ to be

$$\tilde{f}(s, t) = f(s, t) + \epsilon \cdot \nu(s, t),$$

where $\nu$ is the Gauss normal vector. Show that the principal curvatures of $f$ and $\tilde{f}$ are related by $\tilde{\lambda}_i = \lambda_i / (1 - \epsilon \lambda_i)$ ($i = 1, 2$). You may assume that $\epsilon$ is as small as needed for the argument.