Exercise 1. Let $M$ be smooth manifold embedded in $\mathbb{R}^N$. Show that for a residual subset of the dual space of $\mathbb{R}^N$ the restriction on a linear functional to $M$ is a Morse function.

Exercise 2. Show that every Morse function on a closed orientable 2 dimensional manifold of genus $g$ has at least $2g + 2$ critical points.

Exercise 3. Show that the space of $n \times n$ symmetric matrices with at least one eigenvalue of multiplicity greater than one is a stratified space with the stratum stratum of large dimension being of codimension three.

Exercise 4. Show that every real vector bundle $\xi$ of rank $k$ over a manifold $X$ of dimension $d$ can be pulled back from the grassmanian $Gr_k(\mathbb{R}^{k+d})$.

Hint: If $\xi$ is pulled back then there is a fiberwise injective map

$$\xi \to \mathcal{e}^{k+d}$$

Use parameteric transeversality to find the codimension of the set where a fiberwise linear map

$$\xi \to \mathcal{e}^{k+d}$$

is not of full rank.