Exercise 1. Let

\[ f(z_0, z_1, z_2, \ldots, z_n) = \frac{\sum_{i=0}^{n}(i + 1)|z_i|^2}{\sum_{i=0}^{n}|z_i|^2} \]

This defines a function on the projective space \( \mathbb{CP}^n \). Find the critical points of this function and compute their indices. Use the Morse complex to compute the homology of \( \mathbb{CP}^n \).

Exercise 2. Find a good cover of a surface of genus \( g \) and compute the \( Č \)ech cohomology of this cover.

Exercise 3. Prove the following formulae where \( \alpha \) and \( \beta \) are forms and \( v \) and \( w \) are vector fields.

1. \( \iota_v(\alpha \wedge \beta) = (\iota_v \alpha) \wedge \beta + (-1)^{|\alpha|}\alpha \wedge (\iota_v \beta) \)

2. \( [\mathcal{L}_v, \mathcal{L}_w] = \mathcal{L}_{[v,w]} \)

Exercise 4. Find a formula for \( \mathcal{L}_v \alpha \) in local coordinates.

Exercise 5. Show that if \( \mathcal{G} \) is a good cover (all intersections are contractible or empty) and \( \mathcal{U} \) is a refinement of \( \mathcal{G} \) then

\[ \check{H}(\mathcal{U}) \equiv \check{H}(\mathcal{G}) \]