Open Problem 3.3  Let $G = (V, E, W)$ be a graph and $k$ a positive integer, is the following true?

$$\rho_G(k) \leq \text{polylog}(k) \sqrt{\lambda_k}. \quad (30)$$

We note that (30) is known not to hold if we ask that the subsets form a partition (meaning that every vertex belongs to at least one of the sets) [LRTV12]. Note also that no dependency on $k$ would contradict the Small-Set Expansion Hypothesis above.

Reference
