Due date: \textbf{12/5/13 (no need to turn in)}
Collaboration on homework is encouraged, but you should think through the problems yourself before discussing them with other people. \textbf{You must write your solution in your own words. Make sure to list all your collaborators.}

\textbf{Part A}

Part A has problems that straightforwardly follow from the definition. Use this part as an opportunity to get used to the concepts and definitions.

\textbf{Problem A-1.} Verify that the given processes solve the given corresponding stochastic differential equations.
(a) \( X_t = \exp(B_t) \) solves
\[ dX_t = \frac{1}{2} X_t \, dt + X_t \, dB_t. \]
(b) \( X_t = \frac{B_t}{1+t} \) solves
\[ dX_t = -\frac{1}{1+t} X_t \, dt + \frac{1}{1+t} \, dB_t. \]
(c) \( X_t = \sin B_t \) solves
\[ dX_t = -\frac{1}{2} X_t \, dt + \sqrt{1 - X_t^2} \, dB_t. \]

\textbf{Problem A-2.} Let \( a > 0 \) and suppose that
\[ dX_t = \frac{1}{3} X_t^{1/3} \, dt + X_t^{2/3} \, dB_t. \]
Show that
\[ X_t = (a^{1/3} + \frac{1}{3} B_t)^3 \quad t \geq 0, \]
solves the SDE given above when the initial condition is given as \( X_0 = a \).

\textbf{Problem A-3.} (Vasicek interest rate model) Prove that
\[ R(t) = e^{-\beta t} R(0) + \frac{\alpha}{\beta} (1 - e^{-\beta t}) + \sigma e^{-\beta t} \int_0^t e^{\beta s} \, dB_s \]
solves the SDE
\[ dR(t) = (\alpha - \beta R(t)) \, dt + \sigma dB_t. \]