Case Study: Factor Modeling

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1 Load R Libraries and Federal Reserve Data

The R script (“fm_casestudy_7_0.r”) collects daily US Treasury yield data from FRED, the Federal Reserve Economic Database, and stores them in the R workspace “casestudy_7.RData”.

The following commands re-load the data. The series are constant-maturity yields on US Treasury bills, notes and bonds.

```r
> source("fm_casestudy_0_InstallOrLoadLibraries.r")
> # load the R workspace created by the script file
> # fm_casestudy_7_0.r
> dbnames0<-load(file="fm_casestudy_7_0.RData")
> print(dbnames0)
[1] "fred.data00" "date.start" "date.end"
>
>
> library ("graphics")
> library("quantmod")
> fred.data00.0<-na.omit(fred.data00)
> head(fred.data00.0)

<table>
<thead>
<tr>
<th></th>
<th>DGS3MO</th>
<th>DGS6MO</th>
<th>DGS1</th>
<th>DGS2</th>
<th>DGS3</th>
<th>DGS5</th>
<th>DGS7</th>
<th>DGS10</th>
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<td>6.00</td>
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<td>6.35</td>
<td>6.42</td>
<td>6.58</td>
<td>6.52</td>
<td>6.82</td>
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</tbody>
</table>
>
> tail(fred.data00.0)

<table>
<thead>
<tr>
<th></th>
<th>DGS3MO</th>
<th>DGS6MO</th>
<th>DGS1</th>
<th>DGS2</th>
<th>DGS3</th>
<th>DGS5</th>
<th>DGS7</th>
<th>DGS10</th>
<th>DGS20</th>
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</thead>
<tbody>
<tr>
<td>2013-05-23</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>0.26</td>
<td>0.42</td>
<td>0.91</td>
<td>1.40</td>
<td>2.02</td>
<td>2.82</td>
</tr>
<tr>
<td>2013-05-24</td>
<td>0.04</td>
<td>0.07</td>
<td>0.12</td>
<td>0.26</td>
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<td>0.13</td>
<td>0.29</td>
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<td>1.02</td>
<td>1.53</td>
<td>2.15</td>
<td>2.95</td>
</tr>
<tr>
<td>2013-05-29</td>
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<td>0.08</td>
<td>0.14</td>
<td>0.30</td>
<td>0.49</td>
<td>1.02</td>
<td>1.51</td>
<td>2.13</td>
<td>2.91</td>
</tr>
<tr>
<td>2013-05-30</td>
<td>0.04</td>
<td>0.07</td>
<td>0.13</td>
<td>0.31</td>
<td>0.49</td>
<td>1.01</td>
<td>1.51</td>
<td>2.13</td>
<td>2.92</td>
</tr>
<tr>
<td>2013-05-31</td>
<td>0.04</td>
<td>0.07</td>
<td>0.14</td>
<td>0.30</td>
<td>0.52</td>
<td>1.05</td>
<td>1.55</td>
<td>2.16</td>
<td>2.95</td>
</tr>
</tbody>
</table>
>
> fred.data00.diff=zoo(diff((coredata(fred.data00.0)))), order.by=time(fred.data00.0)[-1])
> par(mfcol=c(1,1))
> ts.plot(as.ts(fred.data00.0),col=rainbow(0:NCOL(fred.data00.0)),bg="black",
> main="FRED Rates Data: 2001-2013\n(Period 1:2001-2005)"
> legend(x=2880,y=7,
>
```
```R
+ legend=dimnames(fred.data00.0)[[2]],
+ lty=rep(1,times=NCOL(fred.data00.0)),
+ col=rainbow(NCOL(fred.data00.0)),
+ cex=0.70)
> for (year0 in c(1999:2011)){
+   # year0<-2000
+   abline(v=sum(time(fred.data00.0)< as.Date(paste(as.character(year0),"-12-31",sep=""))))
+ }
> for (year0 in c(2000:2005)){
+   # year0<-2000
+   abline(v=sum(time(fred.data00.0)< as.Date(paste(as.character(year0),"-12-31",sep="")))),
+         col=3,lwd=2)
+ }
```
FRED Rates Data: 2001–2013
(Period 1:2001–2005)
2 US Treasury Yields: 2001-2005

> fred.data00.diff.period1<-window(fred.data00.diff, start=as.Date("2001-01-01"),
+   end=as.Date("2005-12-31"))
> period1.name0<="(2001-2005)"
> ts.plot(as.ts(fred.data00.0["2001::2005-12"]),col=rainbow(NCOL(fred.data00.0)),bg="black",
+   main=paste("FRED Data: Rates", period1.name0,sep="\n"))
> #
>
2.1 Means/Volatilities/Correlations of Yield Changes

```r
> yieldchanges.period1 <- fred.data00.diff.period1
> # Means and St Deviations of the yield changes
> yieldchanges.mean <- apply(yieldchanges.period1, 2, mean)
> yieldchanges.vol <- sqrt(apply(yieldchanges.period1, 2, var))
> print( round (data.frame(}
```
daily.mean = yieldchanges.mean,
+ daily.vol = yieldchanges.vol, digits=4)

daily.mean daily.vol
DGS3MO -0.0015 0.0384
DGS6MO -0.0011 0.0384
DGS1 -0.0008 0.0486
DGS2 -0.0006 0.0663
DGS3 -0.0006 0.0698
DGS5 -0.0005 0.0692
DGS7 -0.0006 0.0666
DGS10 -0.0006 0.0627
DGS20 -0.0008 0.0559

> # On average, all the yields declined from 2001-2005.
> # The yield with the highest standard deviation is the 3-year.
> # Correlations between yield changes
> yieldchanges.cor = cor(fred.data00.diff.period1)
> print(round(yieldchanges.cor, digits=3))

DGS3MD DGS6MD DGS1 DGS2 DGS3 DGS5 DGS7 DGS10 DGS20
DGS3MD 1.000 0.813 0.638 0.479 0.443 0.395 0.341 0.312 0.236
DGS6MD 0.813 1.000 0.864 0.698 0.654 0.595 0.542 0.502 0.414
DGS1 0.638 0.864 1.000 0.876 0.835 0.781 0.735 0.697 0.612
DGS2 0.479 0.698 0.876 1.000 0.972 0.920 0.883 0.843 0.762
DGS3 0.443 0.654 0.835 0.972 1.000 0.956 0.923 0.889 0.813
DGS5 0.395 0.595 0.781 0.920 0.956 1.000 0.974 0.953 0.894
DGS7 0.341 0.542 0.735 0.883 0.923 0.974 1.000 0.979 0.942
DGS10 0.312 0.502 0.697 0.843 0.889 0.953 0.979 1.000 0.960
DGS20 0.236 0.414 0.612 0.762 0.813 0.894 0.942 0.960 1.000

The closer the tenors of the yields, the higher the correlations.
> par(mfcol=c(2,1))
> barplot(yieldchanges.mean,
+   main=paste("US Treasury Yield Changes",period1.name0,sep="\n"),
+   ylab="Daily Mean")
> barplot(yieldchanges.vol,
+   main=paste("US Treasury Yield Changes",period1.name0,sep="\n"),
+   ylab="Daily Volatilities")

US Treasury Yield Changes

Daily Mean

-0.0014 -0.0004
DGS3MO DGS1 DGS3 DGS7 DGS20

US Treasury Yield Changes

Daily Volatilities

0.00 0.03 0.06
DGS3MO DGS1 DGS3 DGS7 DGS20
> pairs(yieldchanges.period1[,1:5])
> pairs(yieldchanges.period1[,6:9])
2.2 Principal Components Analysis

```r
> options(width=80)
> obj.princomp0<-princomp(coredata(fred.data00.diff.period1))
> print(summary(obj.princomp0))

Importance of components:

<table>
<thead>
<tr>
<th>Comp.</th>
<th>Standard deviation</th>
<th>Proportion of Variance</th>
<th>Cumulative Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1618033</td>
<td>0.8494434</td>
<td>0.8494434</td>
</tr>
<tr>
<td>2</td>
<td>0.0532346</td>
<td>0.0919483</td>
<td>0.94139169</td>
</tr>
<tr>
<td>3</td>
<td>0.0298834</td>
<td>0.0289747</td>
<td>0.97036644</td>
</tr>
<tr>
<td>4</td>
<td>0.01918045</td>
<td>0.01193650</td>
<td>0.98230294</td>
</tr>
<tr>
<td>5</td>
<td>0.013461432</td>
<td>0.005879525</td>
<td>0.988182464</td>
</tr>
<tr>
<td>6</td>
<td>0.011196400</td>
<td>0.004067397</td>
<td>0.992249861</td>
</tr>
<tr>
<td>7</td>
<td>0.009568498</td>
<td>0.002970621</td>
<td>0.995220482</td>
</tr>
<tr>
<td>8</td>
<td>0.009009979</td>
<td>0.002633948</td>
<td>0.997854430</td>
</tr>
<tr>
<td>9</td>
<td>0.008131889</td>
<td>0.002145570</td>
<td>1.000000000</td>
</tr>
</tbody>
</table>

> #Note:
> # The first principal component variable explains 84.9% of the total variability
> # The second principal component variable explains 9.2% of the total variability
> # The first three principal component variables explain 97.0% of the total variability
```

```r
> par(mfcol=c(2,1))
> # The screeplot is a barplot of the variances of the Principal Component Variables
> # It provides a graphical display of the summary.
> screeplot(obj.princomp0,main=paste("Scree Plot PCA US Treasury Yield Changes", period1.name0,sep="\n") )
> # The plot below the scree plot compares the standard deviations (daily) of the
> # yield changes for US Treasury yields to those for the principal component variables.
> #
> # The principal components variables are translations and orthogonal rotations of the
> # original variables. The first principal component variable has the highest variance
> # and the last has the smallest.
> yieldchanges.pcvars.vol<-sqrt(apply(obj.princomp0$scores,2,var))
> #
> ts.plot(cbind(
+ as.matrix(yieldchanges.vol),
+ as.matrix(yieldchanges.pcvars.vol)), type="b", col=c(3,4),
+ xlab="Variables: US Treasury (Green) and PC Vars (Blue)",
+ main="Volatility of Yield Changes")
```

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Volatility of Yield Changes

Variables: US Treasury (Green) and PC Vars (Blue)
The loadings component specifies the principal components variables:

```r
> print(round(obj.princomp0$loadings[,1:9],digits=4))

                  Comp.1    Comp.2    Comp.3    Comp.4    Comp.5    Comp.6    Comp.7    Comp.8    Comp.9
DGS3MD  0.1088  -0.5225  0.5434  -0.5408  0.2107  0.2865  -0.0041  -0.0093  -0.0291
DGS6MD  0.1581  -0.4817  0.2508  0.2555 -0.2581 -0.7343  0.0634  0.0473  0.0524
DGS1    0.2531  -0.4107  0.6687  0.0306  0.5244 -0.1674 -0.0914 -0.0712
DGS2    0.3905  -0.2071  0.4750 -0.0670  0.2555  0.0473  0.0473  0.0524
DGS3    0.4197  -0.0873 -0.4022  0.2965  0.0657 -0.2045 -0.5551  0.4197  -0.1900
DGS5    0.4206  0.1083  0.0311  0.2101 -0.5392  0.1841 -0.1817  0.4776  0.3455
DGS7    0.4009  0.2289  0.1475 -0.0204 -0.2167  0.0356  0.3865  0.0047 -0.7530
DGS10   0.3697  0.2837  0.2710  0.0666 -0.0173  0.0351  0.2898 -0.6191  0.4859
DGS20   0.3101  0.3621  0.3944  0.2316  0.5286 -0.1515 -0.3749  0.3472 -0.0033
```

The first principal component variable is a weighted average of all the yield changes, giving highest weight to the 5-year tenor.
The second principal component measures the spread between long-tenor and short-tenor yields.
The third principal component variable measures the curvature in the yield curve.
The last (9th) principal component variable measures the ‘hedge’ of the 7-year yield with the 5-year and 10-year yields.
US Treasury Yield Changes: PCA
Loadings: PC1

Loadings: PC2
> par(mfcol=c(2,1))
> barplot(obj.princomp0$loadings[,3],
+    main="US Treasury Yield Changes: PCA\nLoadings: PC3")
> barplot(obj.princomp0$loadings[,9],
+    main="Loadings: PC9")
> # PC9 has the smallest variability
> # It is dominated by the matching of 7-Year yield changes by
> # weighted average of 5-Year and 10-Year
>
US Treasury Yield Changes: PCA

Loadings: PC3

Loadings: PC9
The principal component variables are uncorrelated. This is evident from the correlation matrix of the variables and the pairs plot:

```r
> print(round(cor(obj.princomp0$scores)))
```

```
          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
Comp.1  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
Comp.2  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
Comp.3  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0  0.0
Comp.4  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0  0.0
Comp.5  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0  0.0
Comp.6  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  0.0
Comp.7  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0
Comp.8  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0
Comp.9  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.0
```

```r
> pairs(obj.princomp0$scores[,c(1:3,9)])
```

> 17
Plotting the cumulative scores of the principal components variables displays the evolution of these factors characterizing the variability in the treasury yield data.

```r
> par(mfcol=c(1,1))
> ts.plot(cumsum(obj.princomp0$scores[,1]),
+         main="Cumulative PC Variables\nPC1 (Level-Shifts)",
+         ylab="Cumulative PC1",xlab="Day")
```

Cumulative PC Variables
PC1 (Level–Shifts)
> ts.plot(cumsum(obj.princomp0$scores[,2]),
+   main="Cumulative PC Variables
PC2 (Spread Between Long and Short Maturities)",
+   ylab="Cumulative PC2", xlab="Day")
```r
> ts.plot(cumsum(obj.princomp0$scores[,3]),
  + main="Cumulative PC Variables\nPC3 (Curvature of Term Structure)",
  + ylab="Cumulative PC3", xlab="Day")
```

Cumulative PC Variables
PC3 (Curvature of Term Structure)
```r
> ts.plot(cumsum(obj.princomp0$scores[,9]),
+     main="Cumulative PC Variables\nPC9 (7 Year vs 5 and 10 Year)",
+     ylab="Cumulative PC9",
+     xlab="Day")
> ###########################
> 
```

### Cumulative PC Variables

PC9 (7 Year vs 5 and 10 Year)
2.3 Factor Analysis

The R function `factanal()` performs the maximum-likelihood factor analysis. The output includes the results of the likelihood ratio test for the sufficiency of the given number of factors.

```r
> # The computations lead to rejecting the hypothesis that 4 factors is sufficient.
> print(factanal(yieldchanges.period1, factors=4))

Call:
factanal(x = yieldchanges.period1, factors = 4)

Uniquenesses:
DGS3MO DGS6MO DGS1 DGS2 DGS3 DGS5 DGS7 DGS10 DGS20
0.316 0.005 0.106 0.005 0.026 0.012 0.016 0.017 0.041

Loadings:

<table>
<thead>
<tr>
<th></th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>Factor4</th>
</tr>
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<td>0.815</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGS6MO</td>
<td>0.265</td>
<td>0.947</td>
<td>0.168</td>
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<tr>
<td>DGS1</td>
<td>0.489</td>
<td>0.705</td>
<td>0.395</td>
<td></td>
</tr>
<tr>
<td>DGS2</td>
<td>0.673</td>
<td>0.445</td>
<td>0.586</td>
<td></td>
</tr>
<tr>
<td>DGS3</td>
<td>0.751</td>
<td>0.392</td>
<td>0.501</td>
<td></td>
</tr>
<tr>
<td>DGS5</td>
<td>0.867</td>
<td>0.330</td>
<td>0.330</td>
<td>0.140</td>
</tr>
<tr>
<td>DGS7</td>
<td>0.922</td>
<td>0.272</td>
<td>0.242</td>
<td></td>
</tr>
<tr>
<td>DGS10</td>
<td>0.948</td>
<td>0.235</td>
<td>0.171</td>
<td></td>
</tr>
<tr>
<td>DGS20</td>
<td>0.959</td>
<td>0.154</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SS loadings 4.758 2.671 0.988 0.039
Proportion Var 0.529 0.297 0.110 0.004
Cumulative Var 0.529 0.825 0.935 0.940

Test of the hypothesis that 4 factors are sufficient.
The chi square statistic is 26.76 on 6 degrees of freedom.
The p-value is 0.00016

> # The computations also lead to rejecting the hypothesis that 5 factors is sufficient.
> print(factanal(yieldchanges.period1, factors=5))

Call:
factanal(x = yieldchanges.period1, factors = 5)

Uniquenesses:
DGS3MO DGS6MO DGS1 DGS2 DGS3 DGS5 DGS7 DGS10 DGS20
0.231 0.062 0.005 0.005 0.026 0.010 0.016 0.016 0.042
Loadings:

<table>
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<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>Factor4</th>
<th>Factor5</th>
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<td>0.864</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>DGS6MO</td>
<td>0.278</td>
<td>0.895</td>
<td>0.206</td>
<td>0.130</td>
<td></td>
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<tr>
<td>DGS1</td>
<td>0.479</td>
<td>0.667</td>
<td>0.396</td>
<td>0.405</td>
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</tr>
<tr>
<td>DGS2</td>
<td>0.680</td>
<td>0.420</td>
<td>0.589</td>
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<td></td>
</tr>
<tr>
<td>DGS3</td>
<td>0.758</td>
<td>0.370</td>
<td>0.505</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGS5</td>
<td>0.871</td>
<td>0.309</td>
<td>0.333</td>
<td>0.145</td>
<td></td>
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<tr>
<td>DGS7</td>
<td>0.925</td>
<td>0.251</td>
<td>0.242</td>
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<tr>
<td>DGS10</td>
<td>0.950</td>
<td>0.217</td>
<td>0.167</td>
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<tr>
<td>DGS20</td>
<td>0.959</td>
<td>0.136</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

SS loadings | 4.797 | 2.530 | 1.008 | 0.217 | 0.034 |
Proportion Var | 0.533 | 0.281 | 0.112 | 0.024 | 0.004 |
Cumulative Var | 0.533 | 0.814 | 0.926 | 0.950 | 0.954 |

Test of the hypothesis that 5 factors are sufficient.
The chi square statistic is 4.33 on 1 degree of freedom.
The p-value is 0.0374

A six-factor model cannot be fit to the data. For this period, the structure
of the yield changes is such that a reduced-dimension model for the correlation
structure is rejected by the likelihood ratio test.

3 US Treasury Yields: 2009-2012

> fred.data00.diff.period2<-window(fred.data00.diff, start=as.Date("2009-01-01"),
+ end=as.Date("2013-12-31"))
> period2.name0<="(2009-2013)"
> ts.plot(as.ts(fred.data00.0["2009::2013"],col=rainbow(ncol(fred.data00.0)),bg="black",
+ main=paste("FRED Data: Rates", period2.name0,sep="\n"))
> #
> 

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3.1 Means/Volatilities/Correlations of Yield Changes

> yieldchanges.period2<-fred.data00.diff.period2
> # Means and St Deviations of the yield changes
> yieldchanges.mean<-apply(yieldchanges.period2,2,mean)
> yieldchanges.vol<-sqrt(apply(yieldchanges.period2,2,var))
> print(round(data.frame(}
> # On average, all the yields declined since 2009.  
> # The yields with the highest standard deviations are the 7, 10, and 20 year.  
> 
> > # Correlations between yield changes  
> yieldchanges.cor<-cor(fred.data00.diff.period2)  
> print(round(yieldchanges.cor, digits=3))  

The closer the tenors of the yields, the higher the correlations.
> par(mfcol=c(2,1))
> barplot(yieldchanges.mean,
+     main=paste("US Treasury Yield Changes",period2.name0,sep="\n"),
+     ylab="Daily Mean")
> barplot(yieldchanges.vol,
+     main=paste("US Treasury Yield Changes",period2.name0,sep="\n"),
+     ylab="Daily Volatilities")
The yields on average fell during the period from 2009-2013. The volatility of the yield changes is highest for tenors greater than 5 years (7, 10, and 20 years).
> pairs(yieldchanges.period2[,1:5])
> pairs(yieldchanges.period2[,6:9])

![Scatter plot matrix for yield changes](image)
### 3.2 Principal Components Analysis

```r
options(width=80)
obj.princomp0<-princomp(coredata(fred.data00.diff.period2))
print(summary(obj.princomp0))
```

#### Importance of components:

<table>
<thead>
<tr>
<th>Comp</th>
<th>Standard deviation</th>
<th>Proportion of Variance</th>
<th>Cumulative Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comp.1</td>
<td>0.1339363</td>
<td>0.885611</td>
<td>0.885611</td>
</tr>
<tr>
<td>Comp.2</td>
<td>0.03706452</td>
<td>0.067821</td>
<td>0.95343190</td>
</tr>
<tr>
<td>Comp.3</td>
<td>0.01778919</td>
<td>0.015623</td>
<td>0.96905471</td>
</tr>
<tr>
<td>Comp.4</td>
<td>0.01391865</td>
<td>0.009564</td>
<td>0.984624344</td>
</tr>
<tr>
<td>Comp.5</td>
<td>0.011029487</td>
<td>0.006005</td>
<td>0.984624344</td>
</tr>
<tr>
<td>Comp.6</td>
<td>0.009714252</td>
<td>0.004659</td>
<td>0.989283047</td>
</tr>
</tbody>
</table>

#### Note:
- The first principal component variable explains 88.6% of the total variability.
- The second principal component variable explains 6.8% of the total variability.
- The first three principal component variables explain 95.9% of the total variability.

#### When compared to the period 2001-2005, the first principal component variable explains modestly more of the variability 88.6% vs 84.9%.

Volatility of Yield Changes

Variables: US Treasury (Green) and PC Vars (Blue)
> # The loadings component specifies the principal components variables:
> print(round(obj.princomp0$loadings[,1:9],digits=4))

<table>
<thead>
<tr>
<th>Comp.1</th>
<th>Comp.2</th>
<th>Comp.3</th>
<th>Comp.4</th>
<th>Comp.5</th>
<th>Comp.6</th>
<th>Comp.7</th>
<th>Comp.8</th>
<th>Comp.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGS3MO</td>
<td>-0.0109</td>
<td>-0.0453</td>
<td>0.3093</td>
<td>0.2362</td>
<td>0.5371</td>
<td>0.1003</td>
<td>-0.1827</td>
<td>-0.3524</td>
</tr>
<tr>
<td>DGS6MO</td>
<td>-0.0260</td>
<td>-0.0775</td>
<td>0.2938</td>
<td>0.4393</td>
<td>0.0478</td>
<td>-0.1151</td>
<td>0.0822</td>
<td>0.4843</td>
</tr>
<tr>
<td>DGS1</td>
<td>-0.0598</td>
<td>-0.1828</td>
<td>0.3444</td>
<td>0.3222</td>
<td>-0.3242</td>
<td>-0.6821</td>
<td>-0.1406</td>
<td>-0.3046</td>
</tr>
<tr>
<td>DGS2</td>
<td>-0.2059</td>
<td>-0.5100</td>
<td>0.4134</td>
<td>-0.3489</td>
<td>-0.4829</td>
<td>0.3778</td>
<td>-0.0515</td>
<td>0.1593</td>
</tr>
<tr>
<td>DGS3</td>
<td>-0.2992</td>
<td>-0.4904</td>
<td>0.0520</td>
<td>-0.1910</td>
<td>0.5624</td>
<td>-0.1072</td>
<td>0.1746</td>
<td>-0.4448</td>
</tr>
<tr>
<td>DGS5</td>
<td>-0.4340</td>
<td>-0.2815</td>
<td>-0.2780</td>
<td>0.1083</td>
<td>0.2730</td>
<td>-0.1627</td>
<td>-0.2200</td>
<td>0.5624</td>
</tr>
<tr>
<td>DGS7</td>
<td>-0.4848</td>
<td>0.0125</td>
<td>-0.4418</td>
<td>0.2468</td>
<td>-0.4281</td>
<td>0.0542</td>
<td>0.5455</td>
<td>-0.1341</td>
</tr>
<tr>
<td>DGS10</td>
<td>-0.4782</td>
<td>0.2616</td>
<td>-0.0577</td>
<td>0.1008</td>
<td>-0.0742</td>
<td>0.1890</td>
<td>-0.7076</td>
<td>-0.2611</td>
</tr>
<tr>
<td>DGS20</td>
<td>-0.4601</td>
<td>0.5568</td>
<td>0.5001</td>
<td>-0.2681</td>
<td>0.1550</td>
<td>-0.1169</td>
<td>0.2890</td>
<td>0.1167</td>
</tr>
</tbody>
</table>

The first principal component variable is a weighted average of all the yield changes, giving greater weights to the longer tenors. In contrast to the 2001-2005 period, the short-tenor yields have low weights. This is due to the low-magnitude yields at the short tenors which consequently have very low variability (and thus little contribution to the first principal component variable). The highest weight to the 5-year tenor.

The second principal component measures the spread between long-tenor and medium-tenor yields.

The third principal component variable measures the curvature in the yield curve.

The last (9th) principal component variable might be interpretable as the ‘hedge’ of the 6-month yield with the 3-month and 5-year yields.

> par(mfcol=c(2,1))
> barplot(obj.princomp0$loadings[,1],
> main="US Treasury Yield Changes: PCA\n Loadings: PC1")
> barplot(obj.princomp0$loadings[,2],
> main="Loadings: PC2")
>
US Treasury Yield Changes: PCA
Loadings: PC1

-0.4  0.0  0.4

DGS3MO  DGS1  DGS3  DGS7  DGS20

Loadings: PC2

-0.4  0.0  0.4

DGS3MO  DGS1  DGS3  DGS7  DGS20
> par(mfcol=c(2,1))
> barplot(obj.princomp$loadings[,3],
> + main="US Treasury Yield Changes: PCA\nLoadings: PC3")
> barplot(obj.princomp$loadings[,9],
> + main="Loadings: PC9")
> # PC9 has the smallest variability
> # It is dominated by the matching of 7-Year yield changes by
> # weighted average of 5-Year and 10-Year
The principal component variables are uncorrelated. This is evident from the correlation matrix of the variables and the pairs plot:

```r
> print(round(cor(obj.princomp0$scores)))

       Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8 Comp.9
Comp.1  1   0   0   0   0   0   0   0   0
Comp.2  0   1   0   0   0   0   0   0   0
Comp.3  0   0   1   0   0   0   0   0   0
Comp.4  0   0   0   1   0   0   0   0   0
Comp.5  0   0   0   0   1   0   0   0   0
Comp.6  0   0   0   0   0   1   0   0   0
Comp.7  0   0   0   0   0   0   1   0   0
Comp.8  0   0   0   0   0   0   0   1   0
Comp.9  0   0   0   0   0   0   0   0   1

> pairs(obj.princomp0$scores[,c(1:3,9)])
```
Plotting the cumulative scores of the principal components variables displays the evolution of these factors characterizing the variability in the treasury yield data.

```r
> par(mfcol=c(1,1))
> ts.plot(cumsum(obj.princomp0$scores[,1]),
> + main="Cumulative PC Variables\nPC1 (Level-Shifts)",
> + ylab="Cumulative PC1",xlab="Day")
```

Cumulative PC Variables
PC1 (Level–Shifts)
> ts.plot(cumsum(obj.princomp0$scores[,2]),
+    main="Cumulative PC Variables
PC2 (Spread Between Long and Short Maturities)",
+    ylab="Cumulative PC2", xlab="Day"

Cumulative PC Variables
PC2 (Spread Between Long and Short Maturities)
> ts.plot(cumsum(obj.princomp0$scores[,3]),
+     main="Cumulative PC Variables\nPC3 (Curvature of Term Structure)",
+     ylab="Cumulative PC3", xlab="Day")
> ts.plot(cumsum(obj.princomp0$scores[,9]),
+     main="Cumulative PC Variables\nPC9 (7 Year vs 5 and 10 Year)",
+     ylab="Cumulative PC9",
+     xlab="Day")
>

Cumulative PC Variables
PC9 (7 Year vs 5 and 10 Year)
3.3 Factor Analysis

The r function `factanal()` performs the maximum-likelihood factor analysis. The output includes the results of the likelihood ratio test for the sufficiency of the given number of factors.

```r
> print(factanal(yieldchanges.period2,factors=3))

Call:
factanal(x = yieldchanges.period2, factors = 3)

Uniquenesses:
DGS3MO DGS6MO DGS1 DGS2 DGS3 DGS5 DGS7 DGS10 DGS20
0.677 0.487 0.386 0.140 0.044 0.023 0.033 0.006 0.062

Loadings:
   Factor1 Factor2 Factor3
DGS3MO 0.567        
DGS6MO 0.116 0.200 0.677
DGS1  0.232 0.496 0.561
DGS2  0.436 0.759 0.307
DGS3  0.589 0.752 0.211
DGS5  0.771 0.603 0.141
DGS7  0.866 0.456 0.101
DGS10 0.937 0.314 0.131
DGS20 0.941 0.175 0.144

            Factor1 Factor2 Factor3
SS loadings 3.712 2.128 1.302
Proportion Var 0.412 0.236 0.145
Cumulative Var 0.412 0.649 0.793

Test of the hypothesis that 3 factors are sufficient.
The chi square statistic is 231.03 on 12 degrees of freedom.
The p-value is 1.22e-42

> print(factanal(yieldchanges.period2,factors=4))

Call:
factanal(x = yieldchanges.period2, factors = 4)

Uniquenesses:
DGS3MO DGS6MO DGS1 DGS2 DGS3 DGS5 DGS7 DGS10 DGS20
0.651 0.360 0.412 0.022 0.061 0.005 0.033 0.005 0.062

Loadings:
   Factor1 Factor2 Factor3 Factor4
DGS3MO 0.567        
...
<table>
<thead>
<tr>
<th></th>
<th>Factor1</th>
<th>Factor2</th>
<th>Factor3</th>
<th>Factor4</th>
</tr>
</thead>
<tbody>
<tr>
<td>DGS3MO</td>
<td>0.588</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DGS6MO</td>
<td>0.119</td>
<td>0.200</td>
<td>0.765</td>
<td></td>
</tr>
<tr>
<td>DGS1</td>
<td>0.246</td>
<td>0.540</td>
<td>0.486</td>
<td></td>
</tr>
<tr>
<td>DGS2</td>
<td>0.435</td>
<td>0.861</td>
<td>0.211</td>
<td></td>
</tr>
<tr>
<td>DGS3</td>
<td>0.627</td>
<td>0.701</td>
<td>0.178</td>
<td>0.152</td>
</tr>
<tr>
<td>DGS5</td>
<td>0.803</td>
<td>0.537</td>
<td>0.140</td>
<td>0.203</td>
</tr>
<tr>
<td>DGS7</td>
<td>0.887</td>
<td>0.400</td>
<td>0.104</td>
<td></td>
</tr>
<tr>
<td>DGS10</td>
<td>0.947</td>
<td>0.291</td>
<td>0.116</td>
<td></td>
</tr>
<tr>
<td>DGS20</td>
<td>0.938</td>
<td>0.182</td>
<td>0.117</td>
<td>-0.104</td>
</tr>
</tbody>
</table>

SS loadings 3.867 2.131 1.302 0.089
Proportion Var 0.430 0.237 0.145 0.010
Cumulative Var 0.430 0.666 0.811 0.821

Test of the hypothesis that 4 factors are sufficient.
The chi square statistic is 5.23 on 6 degrees of freedom.
The p-value is 0.515

For the period since 2009, the yield changes can be described by a 4-factor model.

The uniquenesses are the specific variances of the component yield changes; note the high value for the 3-month yield changes (DGS3MO) as compared with the other tenors.