18.S096: Topics in Mathematics with Applications in Finance

Instructors: Dr. Peter J Kempthorne, Dr. Choongbum Lee, Dr. Vasily Strela, Dr. Jake Xia
18.S096: Key Facts

• The purpose of the class is to expose undergraduate and graduate students to the mathematical concepts and techniques used in the financial industry.

• Mathematics lectures will be mixed with lectures illustrating the corresponding application in the financial industry.

• MIT mathematicians will teach the mathematics part while industry professionals will give the lectures on applications in finance.

• We plan to organize an optional field trip to visit Morgan Stanley offices in New York on October 15.

• Prerequisites: 18.01, 18.02, 18.03 (or approval of instructor), 18.05 or 18.440, 18.06

• Prior knowledge of economics or finance is not required but may be helpful for some lectures.

• The class will have problem sets for each math lecture (due in 2 weeks after assignment) and a final paper on a math finance topic of student’s choice.

• There will be no exams. The final grade will be 75% based on the homework and 25% on final paper.
How are Financial Products Traded?

Introduction of basic knowledge of financial markets and quantitative analysis

Jake Xia
MIT Course 18.S096
September 5, 2013

The views represented herein are the author’s own views and do not necessarily represent the views of his past and current employers.
The Story of
Pi,
Alpha,
Beta,
Delta,
Gamma,
Theta,

and

Vega
Markets, Exchanges, OTC, Regions

Money, Currencies

Stocks, Equity Indices, IPOs, Primary/Secondary

Loans, Bonds, Credit Products

Commodity

Real Estate, Mortgages, ABS

Derivatives, Structured Products
Banks: deposits, commercial lending

Investment Banks:

Fixed Income

Equity

IBD, Corporate Finance

Asset Management

Wealth Management
Zero-sum game?

Investors—have money, need return

Borrowers—need money, have risks

Trade between lender and borrower is still the main driving force.
Market Participants:

Dealers, Market Makers, Brokers

Individuals, Retail Investors

Mutual Funds, Insurance Companies, Pension Funds, Asset Managers, SWFs

Hedge Funds (various strategies), PE

Governments, Policy Makers

Corporate Hedgers, Liability Management
Trader Types:

1. Hedger

2. Market Maker, bid and offer

3. Proprietary Trader
   Fund Portfolio Manager
   beta and alpha, linear regression of two time series: return of assets a and b:
   \[ R(a) = \alpha + \beta \times R(b) \]
Hedging Trade Examples:

Bond issuer hedge interest rate exposures, hedge of payables.
Example: 10yr Samurai bonds issued in JPY by Australia corporates. Fixed rate difference of 4.5% - 1% = 3.5%. Need receive JPY and pay AUD to hedge. FX, swaps and cross-currency basis.

Corporates/producers hedge revenue.
Currency hedge of future receivables

Importer and exporter hedge FX

Deal contingent hedges
Market Making Examples:

Simple product bid/offer, price transparency, liquidity

Inventory of risks, manage a dealer book, balancing Greeks: delta, gamma, theta, vega, tail risks

Value at Risk (VaR), Capital, Balance Sheet, Risk Weight Assets
Proprietary Trading Examples:
Directional, long, short,
Arbitrage
Value, Relative Value
Systematic, momentum, statistical arbitrage
Fundamental Analysis. Global Macro
Special situations, distressed
Financial Mathematics:

1. Pricing Models:
   relative value, arbitrage-free.

2. Risk Management:
   human risk aversion, greed/fear.

3. Trading Strategies:
   “Holy Grail” strategy, perpetual motion machine, Robo-Trader.
Risk aversion:

Choice A: 80% chance to lose $500, 20% chance to win $500.
Choice B: 100% chance to lose $280.

Which choice?
Now

Choice A: 80% chance to win $500, 20% chance to lose $500.
Choice B: 100% chance to win $280.

Which Choice?

Control factor. Marginal utility.
Do people always learn from their experiences?

Long market cycle and short memory
Historical extrapolation.
Deterministic vs statistic.
Over-simplification.
Home work assignment (optional):

Make your own financial glossary (the terminology you don’t understand) by searching the web or reading the materials on the course website.
Project 1 – Noisy Delta

• Monte Carlo pricing
• Delta computation with centered difference
• Question: What is the optimal shift size?

Courtesy of Dusan Milijancevic
Project 1 – Noisy Delta

• For the solution we minimize the standard error

\[
E(|f'(x) - \frac{\hat{f}(x+\epsilon) - \hat{f}(x-\epsilon)}{2\epsilon}|^2)
\]

\[
\mathcal{E}_{opt} = \frac{1}{3\sqrt{2c}} \sqrt{\frac{sd(x)}{d + |\Gamma(x)|}}
\]

Courtesy of Dusan Milijancevic
Project 1 – Noisy Delta

![Graph showing Delta values for different shift sizes.]

Courtesy of Dusan Milijancevic
Project 2 – Kalman Filter

- Five brokers quote swap points for USDRUB forward contract at different times
- The aim of the project is to study if Kalman filter can improve accuracy of quotes

<table>
<thead>
<tr>
<th>Timestamp</th>
<th>ICAP</th>
<th>Timestamp</th>
<th>TRADITION</th>
<th>Timestamp</th>
<th>CONT.CAPITAL</th>
<th>Timestamp</th>
<th>TULL PBON</th>
<th>Timestamp</th>
<th>GFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>19/03/13 8:26:07</td>
<td>4718</td>
<td>19/03/13 7:54:26</td>
<td>4730</td>
<td>19/03/13 7:44:30</td>
<td>4730</td>
<td>19/03/13 8:19:10</td>
<td>4730</td>
<td>19/03/13 8:02:00</td>
<td>4670</td>
</tr>
<tr>
<td>19/03/13 8:31:04</td>
<td>4718</td>
<td>19/03/13 7:56:25</td>
<td>4725</td>
<td>19/03/13 8:03:57</td>
<td>4750</td>
<td>19/03/13 9:13:49</td>
<td>4750</td>
<td>19/03/13 8:05:28</td>
<td>4675</td>
</tr>
<tr>
<td>19/03/13 8:37:35</td>
<td>4721</td>
<td>19/03/13 8:01:18</td>
<td>4730</td>
<td>19/03/13 8:40:12</td>
<td>4750</td>
<td>20/03/13 8:22:31</td>
<td>4730</td>
<td>19/03/13 8:05:35</td>
<td>4660</td>
</tr>
<tr>
<td>19/03/13 8:53:35</td>
<td>4723</td>
<td>19/03/13 8:01:25</td>
<td>4725</td>
<td>19/03/13 12:05:01</td>
<td>4750</td>
<td>20/03/13 11:31:38</td>
<td>4720</td>
<td>19/03/13 8:16:55</td>
<td>4685</td>
</tr>
<tr>
<td>19/03/13 9:05:22</td>
<td>4731</td>
<td>19/03/13 8:03:54</td>
<td>4740</td>
<td>19/03/13 12:26:51</td>
<td>4760</td>
<td>21/03/13 6:36:38</td>
<td>4820</td>
<td>19/03/13 9:09:47</td>
<td>4690</td>
</tr>
<tr>
<td>19/03/13 9:20:54</td>
<td>4728</td>
<td>19/03/13 9:46:26</td>
<td>4740</td>
<td>19/03/13 12:27:05</td>
<td>4750</td>
<td>21/03/13 7:50:17</td>
<td>4840</td>
<td>19/03/13 14:09:29</td>
<td>4685</td>
</tr>
<tr>
<td>19/03/13 10:49:28</td>
<td>4726</td>
<td>19/03/13 9:46:30</td>
<td>4740</td>
<td>19/03/13 12:27:23</td>
<td>770</td>
<td>21/03/13 7:52:47</td>
<td>4860</td>
<td>19/03/13 15:53:37</td>
<td>4690</td>
</tr>
<tr>
<td>19/03/13 13:01:00</td>
<td>4728</td>
<td>19/03/13 11:54:01</td>
<td>4740</td>
<td>19/03/13 12:27:25</td>
<td>4760</td>
<td>21/03/13 8:28:24</td>
<td>4850</td>
<td>19/03/13 15:59:17</td>
<td>4695</td>
</tr>
<tr>
<td>19/03/13 13:01:57</td>
<td>4731</td>
<td>19/03/13 12:09:15</td>
<td>4745</td>
<td>19/03/13 12:34:18</td>
<td>4760</td>
<td>21/03/13 8:33:55</td>
<td>4860</td>
<td>19/03/13 16:03:44</td>
<td>4700</td>
</tr>
<tr>
<td>19/03/13 16:02:40</td>
<td>4738</td>
<td>19/03/13 12:33:53</td>
<td>4745</td>
<td>19/03/13 12:34:26</td>
<td>4750</td>
<td>21/03/13 9:37:37</td>
<td>4880</td>
<td>19/03/13 16:11:14</td>
<td>4705</td>
</tr>
<tr>
<td>19/03/13 16:15:52</td>
<td>4758</td>
<td>19/03/13 12:46:25</td>
<td>4750</td>
<td>19/03/13 12:50:08</td>
<td>4760</td>
<td>22/03/13 6:44:31</td>
<td>4775</td>
<td>19/03/13 16:21:07</td>
<td>4710</td>
</tr>
<tr>
<td>20/03/13 7:08:27</td>
<td>4738</td>
<td>19/03/13 13:18:51</td>
<td>4750</td>
<td>19/03/13 13:16:34</td>
<td>4755</td>
<td>22/03/13 8:00:46</td>
<td>4790</td>
<td>20/03/13 5:54:19</td>
<td>4715</td>
</tr>
<tr>
<td>20/03/13 7:32:59</td>
<td>4743</td>
<td>19/03/13 13:24:35</td>
<td>4740</td>
<td>19/03/13 13:16:52</td>
<td>4755</td>
<td>22/03/13 8:04:17</td>
<td>4785</td>
<td>20/03/13 5:54:23</td>
<td>4725</td>
</tr>
<tr>
<td>20/03/13 7:35:10</td>
<td>4733</td>
<td>19/03/13 13:24:41</td>
<td>4745</td>
<td>19/03/13 13:22:08</td>
<td>4750</td>
<td>22/03/13 8:16:49</td>
<td>4790</td>
<td>20/03/13 7:19:45</td>
<td>4720</td>
</tr>
<tr>
<td>20/03/13 7:40:31</td>
<td>4728</td>
<td>19/03/13 13:25:41</td>
<td>4745</td>
<td>19/03/13 13:22:10</td>
<td>4755</td>
<td>26/03/13 7:01:38</td>
<td>4760</td>
<td>20/03/13 7:20:01</td>
<td>4715</td>
</tr>
<tr>
<td>20/03/13 8:19:05</td>
<td>4723</td>
<td>19/03/13 13:40:06</td>
<td>4745</td>
<td>19/03/13 13:22:30</td>
<td>4755</td>
<td>27/03/13 6:35:29</td>
<td>4770</td>
<td>20/03/13 7:20:12</td>
<td>4710</td>
</tr>
</tbody>
</table>

Courtesy of Dusan Milijancevic
Project 2 – Kalman Filter

KF estimate vs real price

Courtesy of Dusan Milijancevic
Disclosures

The information herein has been prepared solely for informational purposes and is not an offer to buy or sell or a solicitation of an offer to buy or sell any security or instrument or to participate in any trading strategy. Any such offer would be made only after a prospective participant had completed its own independent investigation of the securities, instruments or transactions and received all information it required to make its own investment decision, including, where applicable, a review of any offering circular or memorandum describing such security or instrument, which would contain material information not contained herein and to which prospective participants are referred. No representation or warranty can be given with respect to the accuracy or completeness of the information herein, or that any future offer of securities, instruments or transactions will conform to the terms hereof. Morgan Stanley and its affiliates disclaim any and all liability relating to this information. Morgan Stanley, its affiliates and others associated with it may have positions in, and may effect transactions in, securities and instruments of issuers mentioned herein and may also perform or seek to perform investment banking services for the issuers of such securities and instruments.

The information herein may contain general, summary discussions of certain tax, regulatory, accounting and/or legal issues relevant to the proposed transaction. Any such discussion is necessarily generic and may not be applicable to, or complete for, any particular recipient's specific facts and circumstances. Morgan Stanley is not offering and does not purport to offer tax, regulatory, accounting or legal advice and this information should not be relied upon as such. Prior to entering into any proposed transaction, recipients should determine, in consultation with their own legal, tax, regulatory and accounting advisors, the economic risks and merits, as well as the legal, tax, regulatory and accounting characteristics and consequences, of the transaction.

Notwithstanding any other express or implied agreement, arrangement, or understanding to the contrary, Morgan Stanley and each recipient hereof are deemed to agree that both Morgan Stanley and such recipient (and their respective employees, representatives, and other agents) may disclose to any and all persons, without limitation of any kind, the U.S. federal income tax treatment of the securities, instruments or transactions described herein and any fact relating to the structure of the securities, instruments or transactions that may be relevant to understanding such tax treatment, and all materials of any kind (including opinions or other tax analyses) that are provided to such person relating to such tax treatment and tax structure, except to the extent confidentiality is reasonably necessary to comply with securities laws (including, where applicable, confidentiality regarding the identity of an issuer of securities or its affiliates, agents and advisors).

The projections or other estimates in these materials (if any), including estimates of returns or performance, are forward-looking statements based upon certain assumptions and are preliminary in nature. Any assumptions used in any such projection or estimate that were provided by a recipient are noted herein. Actual results are difficult to predict and may depend upon events outside the issuer’s or Morgan Stanley’s control. Actual events may differ from those assumed and changes to any assumptions may have a material impact on any projections or estimates. Other events not taken into account may occur and may significantly affect the analysis. Certain assumptions may have been made for modeling purposes only to simplify the presentation and/or calculation of any projections or estimates, and Morgan Stanley does not represent that any such assumptions will reflect actual future events. Accordingly, there can be no assurance that estimated returns or projections will be realized or that actual returns or performance results will not be materially different than those estimated herein. Any such estimated returns and projections should be viewed as hypothetical. Recipients should conduct their own analysis, using such assumptions as they deem appropriate, and should fully consider other available information in making a decision regarding these securities, instruments or transactions. Past performance is not necessarily indicative of future results. Price and availability are subject to change without notice.

The offer or sale of securities, instruments or transactions may be restricted by law. Additionally, transfers of any such securities, instruments or transactions may be limited by law or the terms thereof. Unless specifically noted herein, neither Morgan Stanley nor any issuer of securities or instruments has taken or will take any action in any jurisdiction that would permit a public offering of securities or instruments, or possession or distribution of any offering material in relation thereto, in any country or jurisdiction where action for such purpose is required. Recipients are required to inform themselves of and comply with any legal or contractual restrictions on their purchase, holding, sale, exercise of rights or performance of obligations under any transaction. Morgan Stanley does not undertake or have any responsibility to notify you of any changes to the attached information.

With respect to any recipient in the U.K., the information herein has been issued by Morgan Stanley & Co. International Limited, regulated by the U.K. Financial Services Authority. THIS COMMUNICATION IS DIRECTED IN THE UK TO THOSE PERSONS WHO ARE MARKET COUNTERPARTIES OR INTERMEDIATE CUSTOMERS (AS DEFINED IN THE UK FINANCIAL SERVICES AUTHORITY’S RULES).

ADDITIONAL INFORMATION IS AVAILABLE UPON REQUEST.