Recall from last time:

\[
[\sigma] = \begin{vmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{vmatrix}
\]

Diagonal terms are normal stresses.
Off diagonal terms are shear stresses.

Not as bad as it seems:
1. Linearity
2. Superposition

EXAMPLE: Uniaxial Stress
\[ [\sigma] = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \]

EXAMPLE: Hydrostatic Stress

\[ [\sigma] = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}. \]

Note: \( p \) is negative (compressive stresses are negative by convention)

EXAMPLE: Plane Stress

\[ [\sigma] = \begin{bmatrix} \sigma_a & \tau & 0 \\ \tau & \sigma_b & 0 \\ 0 & 0 & 0 \end{bmatrix}. \]
EXAMPLE: Biaxial Plane Stress

\[
[\sigma] = \begin{bmatrix}
\sigma_a & 0 & 0 \\
0 & \sigma_b & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

EXAMPLE: Shear Plane Stress

\[
[\sigma] = \begin{bmatrix}
0 & \tau & 0 \\
\tau & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Note: The stress at a material point that you see, and its magnitude, depends on the orientation of the coordinates relative to your loading. In other words: the state of stress is a function of the chosen coordinate system.
This is called a "stress transformation"

Equilibrium:

\[ \sum F_x = 0 \]
\[-\sigma_{xx} dydz + (\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx) dydz - \sigma_{yz} dxdz + (\sigma_{yz} + \frac{\partial \sigma_{yz}}{\partial y} dy) dxdydz = 0 \]

So:

\[ \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \]
\[ \sum F_y = 0 \]
\[ \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} \]
\[ \sum M \]
\[ (\sigma_{xy} + \frac{\partial \sigma_{xy}}{\partial x} dx) dxdydz + \sigma_{xy} dxdydz \]
\[ - \sigma_{yz} dxdz dy - (\sigma_{yz} + \frac{\partial \sigma_{yz}}{\partial y} dy) dxdydz dy = 0 \]
\[ \sigma_{xy} - \sigma_{yz} + \frac{\partial \sigma_{xy} dx}{2} - \frac{\partial \sigma_{yz} dy}{2} = 0 \]

For \( dx, dy \rightarrow 0 \):

\[ \sigma_{xy} = \sigma_{yz} \]

This shows that the stress tensor \( \sigma \) is symmetric.

This was derived in plane stress (2-D) but it can be extended to 3-D. Here are the results:
\[
\begin{align*}
\sigma_{xy} &= \sigma_{yz} \\
\sigma_{yz} &= \sigma_{zy} \\
\sigma_{xz} &= \sigma_{zx}
\end{align*}
\]
So:
\[
\begin{bmatrix}
\sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_{zz}
\end{bmatrix}.
\]

Note: 6 independent stress terms due to equilibrium. The stress tensor is symmetric.

Strain:

Uniaxial:
\[
\epsilon(x) = \frac{du(x)}{dx} = \frac{\delta}{L}
\]
\[ \vec{u}(x, y, z) = u_x(x, y, z)\hat{i} + u_y(x, y, z)\hat{j} + u_z(x, y, z)\hat{k} \]

Case 1:

No strain. Rigid body translation.

Case 2:

\[ \vec{u}(x, y, z) = a\hat{i} + b\hat{j} \]

No strain. Rigid body translation.

Case 3:
Isotropic Deformation: Stretches evenly so this is linear.

\[ \ddot{u}(x, y, z) = \frac{\delta x}{L} \hat{i} \]

\[ \frac{du_x}{dx} = \frac{\delta}{L} \]

Define:

\[ \epsilon_{xx} = \frac{du_x}{dx}, \text{ Normal Strain (x face in x direction)} \]

Case 4:

\[ \ddot{u}(x, y, z) = \left( a + \frac{\delta}{L} x \right) \hat{i} \]

\[ \epsilon_{xx} = \frac{du_x}{dx} = \frac{\delta}{L} \]

Note: Rigid body translation does not affect the strain.

Case 5:
\[ \vec{u}(x, y, z) = \frac{\delta}{L} y \hat{j} \]

\[ \epsilon_{yy} = \frac{du_y}{dy} = \frac{\delta}{L} \]

**Case 6:**

**Case 7: Shear Strain**

\[ \vec{u}(x, y, z) = \frac{\delta_1}{L} x \hat{i} + \frac{\delta_2}{L} y \hat{j} \]

\[ \epsilon_{xx} = \frac{du_x}{dx} = \frac{\delta_1}{L} \]

\[ \epsilon_{yy} = \frac{du_y}{dy} = \frac{\delta_2}{L} \]

\[ \vec{u}(x, y, z) = \frac{\delta}{L} y \hat{j} \]

\[ \frac{\partial u_x}{\partial y} = \frac{\delta}{L} = \tan \theta \approx \theta \]
Define:

\( \gamma_{xy} = \theta \)

\[ \bar{u}(x, y, z) = \frac{\delta}{L} \hat{j} \]

\[ \frac{\partial u_y(x)}{\partial x} = \frac{\delta}{L} = \theta \]

\( \gamma_{yx} = \theta \)

\( \gamma_{yx} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \)

Define:

\( \epsilon_{xy} = \epsilon_{yx} = \frac{1}{2} \left[ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right] \)