2.003 Problem Set 6

Assigned: Fri. Mar. 11, 2005
Due: Fri. Mar. 18, 2005, in recitation

**Problem 1** Archive Problem 14.2

**Problem 2** Archive Problem 14.3
Do not solve for the transfer function. Rather, find a differential equation which describes the system in terms of an input $F(t)$ and an output $x(t)$.

**Problem 3** Archive Problem 14.4

**Problem 4** Archive Problem 14.5

**Problem 5** Archive Problem 14.7
In section b), assume that the wheels have a radius $R$.

**Problem 6** Archive Problem 14.8

**Problem 7** This problem considers the system shown below in which two rotational inertias $J_1$ and $J_2$ are connected by a gear train.

\[
\begin{array}{c}
\omega_1 \\
\tau_1 \\
1 \\
J_1 \\
N\\nJ_2 \\
\omega_2
\end{array}
\]

The rotational speed of $J_1$ is $\omega_1$ and the rotational speed of $J_2$ is $\omega_2$. An input source of torque $\tau_1$ is applied to $J_1$ in the direction of $\omega_1$. As shown in the figure, the gear train has a ratio of $N : 1$; that is, $\omega_2 = N\omega_1$.

a) Assume that the input torque source has a constant value of $\tau = \tau_0$. What value of $N$ will maximize the acceleration of the load $\ddot{\omega}_2$?

b) For this acceleration-optimum gear ratio, what is the equivalent inertia of $J_2$ as seen by $J_1$ looking through the gear train? That is, what is the reflected inertia of $J_2$ on the $J_1$ side? How does this compare with $J_1$?

c) The power input to the system is $P_{in} = \tau_1\omega_1$. For the optimum gear ratio calculated above, make a plot of $P_{in}(t)$ assuming that the load starts at rest at $t = 0$. 
Problem 8 This problem focuses on the motor connected to a load shown below. The motor is driven by an input voltage \( V \) in series with a coil resistance \( R \). The motor is assumed ideal, with no energy storage or losses inside the motor. The motor is connected to a load inertia \( J > 0 \) and rotational damper \( b \geq 0 \). The motor shaft and load rotate at angular velocity \( \omega \). The motor applies a torque to the load \( \tau = Ki \) in the direction of \( \omega \); correspondingly, the back emf is \( e = K \omega \).

![Diagram of motor connected to load](image)

a) Write a differential equation describing the system in terms of the input voltage \( V(t) \) and the output speed \( \omega(t) \). Write an equivalent differential equation with input \( V(t) \) and output \( i(t) \).

b) Assume that the system is initially at rest, and that at \( t = 0 \) the input voltage takes a step \( V(t) = V_0 u_s(t) \). Solve for the resulting transient in \( i(t) \) and \( \omega(t) \), and make a plot of these two quantities as a function of time.

c) What are the steady-state values of \( i \) and \( \omega \)? In steady-state, write an expression for the power being dissipated on the mechanical side in the load damper \( b \), and on the electrical side in the resistor \( R \). How much power is being supplied in steady-state by the voltage source? Is this in balance with the dissipation? In this steady-state, how much kinetic energy is stored in the load inertia \( J \)?

d) Make a plot of steady-state load power dissipation as a function of load damping \( b \) for \( b \geq 0 \). What value of \( b \) results in maximum power dissipation in the load? How does this compare with the electrical equivalent damping term \( K^2/R \)? For this maximum power value of \( b \), how much power is being dissipated in the resistor \( R \)?

e) Finally, suppose we allow negative values of the load damping. Note that a negative damper will supply power to the load. For what range of \( b < 0 \) will the system be stable? For what range of \( b < 0 \) will the system be unstable?