2.003 Spring 2003
Quiz 2 - Sample problem Set 2 Solutions

Problem A - RLC circuit analysis

1. \[
\frac{V_o}{V_i} = \frac{1}{LCs^2 + RCs + 1}
\]

2. \[
\omega_n = 2 \pi \times 5000 = 31,400 \frac{r}{s} = \frac{1}{\sqrt{LC}}
\]
\[
L = \frac{1}{\omega_n^2 C} = 0.001 \text{ } H = 1 \text{ } mH
\]
\[
\frac{R}{L} = 2\zeta\omega_n = 2 \times 0.707 \times 31,400
\]
\[
R = 44.4 \Omega
\]

3. There are no zeros, poles at roots of
\[
s^2 + \frac{1000}{0.001} s + \frac{1}{1e - 6 \times 1e - 3} = 0
\]
\[
s_1 \approx -1e6
\]
\[
s_2 \approx -1e3 \text{ dominant pole}
\]
\[
x_{ss} = 1
\]
4. 

5. \( s = -1e3 = s_2 \)

6. 

For circuit with R&C in \( \| \) \( \frac{v_o}{v_i} = \frac{R_2}{R_2 LCs^2 + (R_1 R_2 C + L)s + R_1 + R_2} \)

For circuit with R&C in series \( \frac{v_o}{v_i} = \frac{R_2}{R_2 Cs + 1} \)

Problem B

1. 

\[
T(s) = \frac{K}{s^2 + 20s + K} \\
\omega_n = \sqrt{K} \\
2\zeta\omega_n = 2\sqrt{K} = 20 \\
K = 100
\]

2. 

\[
\frac{V_{out}}{V_1} = \frac{(s + 3)(6s + 1)}{(s + 3)(6s + 1) + (8s + 7)} \\
\frac{V_{out}}{V_2} = \frac{(6s + 1)}{(s + 3)(6s + 1) + (8s + 7)}
\]

3. Solve using superposition

\[
V_{out}(6s^2 + 27s + 10) = V_1(6s^2 + 19s + 3) + V_2(6s + 1) \\
6\dot{V}_{out} + 27\ddot{V}_{out} + 10V_{out} = 6\ddot{V}_1 + 19\dot{V}_1 + 3V_1 + 6(V)_2 + V_2
\]
Problem C
The transfer function for this system is
\[
\frac{\mathcal{X}(s)}{\mathcal{F}(s)} = \frac{1}{m_{eq}s^2 + cs + k}
\]
\[
\omega_n = \sqrt{\frac{k}{m_{eq}}}
\]
\[
2\zeta \omega_n = \frac{c}{m_{eq}}
\]
1. From graph, we measure the following
\[
T \approx 1.0 s \Rightarrow \omega_d = \frac{2\pi}{T} = 6.28 r/s
\]
\[
\omega_d = \omega_n \sqrt{1 - \zeta^2}
\]
\[
M_p \approx 100 \frac{0.75 - 0.5}{0.5} = 50
\]
\[
\zeta = \frac{A}{\sqrt{\pi^2 + A^2}} = 0.215
\]
\[
A = ln \frac{100}{M_p} = 0.693
\]
\[
\omega_n = 6.43 r/s
\]
\[
m_{eq} = \frac{\omega_n^2}{k} = 4.8 \approx 5 kg
\]
\[
c = 2\zeta \omega_n m_{eq} = 13.8 Ns/m \approx 14 Ns/m
\]
Alternately, you could determine $\zeta$ using the log decrement method.

2.
\[
m_{eq} = m + \frac{I}{r^2}
\]
\[
I = 0.5 kg m^2
\]

Problem D
The transfer function for this system is
\[
\frac{\omega(s)}{\phi(s)} = \frac{k}{Js^2 + cs + k}
\]
Thus
\[
\omega_n = \sqrt{\frac{k}{J}}
\]
\[
2\zeta \omega_n = \frac{c}{J}
\]
1. There are a couple of ways to solve this part of the problem. First, you can read $\omega_r = 9 r/s$ and $M_p = 5 dB$ from the bode plot and use the following
relationships

\[ M_p = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} \]
\[ \omega_r = \omega_n \sqrt{1 - 2\zeta^2} \]

to find \( \zeta \approx 0.3 \) and \( \omega_n \approx 10 \, r/s \). Or you can read \( \omega_n = 10 \, r/s \) directly from the phase plot \((\theta = -90^\circ)\)

2. \( k = 1500 \, Nm/r, \, c = 90Nms/r \)

3.

\[ \omega = 1.1 \, r/s \, \theta(t) \approx \sin(1.1t + 0) \]
\[ \omega = 10 \, r/s \, \theta(t) \approx 1.58\sin(10t - \pi/2) \]
\[ \omega = 20 \, r/s \, \theta(t) \approx 0.3\sin(20t - 2.75) \]