Problem Set 10
Out: Tuesday, December 4th, 2007
Due: Wednesday, December 12th, 2007

Free and forced system response

Problem 1. Rack and pinion

Consider the rack and the pinion system shown in Fig.1 below as in Problem 1 of Pset 8. The axis of the pinion is fixed in frictionless bearings. A massless rocket is attached to the circular massless pulley of radius $a$ at a point along its edge as shown in the figure. It exerts a thrust $F(t)$ which remains tangential to the pulley at all times. Assume that the pinion can be modeled as a uniform cylinder of mass $m_2$ and radius $b$ and that the friction between the rack and the horizontal surface can be modeled as viscous damping having a dashpot constant $c$.

a. Find the equilibrium of the system, and linearize the equations of motion about it.

b. Reduce the system to the canonical form with equivalent mass, spring, dashpot, and generalized force. Identify the values of parameters $\omega_n$ and $\zeta$.

c. Determine the conditions on the values of $m_1$, $m_2$, $k$ and $c$ for which the system is under-damped, critically damped and over-damped.

d. For $F(t) = F_o \cos(\omega t)$, determine the particular solution to the ODE. Determine the value of the driving frequency at which the resonance occurs (when $\zeta = 0$).

Figure 1. Rack and pinion

Figure by MIT OpenCourseWare.

Cite as: Sanjay Sarma, Nicholas Makris, Yahya Modarres-Sadeghi, and Peter So, course materials for 2.003J/1.053J Dynamics and Control I, Fall 2007. MIT OpenCourseWare (http://ocw.mit.edu), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Problem 2. Design the rack and pinion system

Constraints on the operation of the system in Problem 1 (about equilibrium) require that the system operates in the underdamped mode, with a maximum overshoot of 15% and a peak time of 1 ms. The mass of the rack is \( m_1 = 50 \text{ kg} \), and that of the pinion is \( m_2 = 20 \text{ kg} \).

a. Find \( k \) and \( c \) required to meet the above specifications.

b. Approximately how long will it take the system to settle?
Problem 3. Gear train

Consider the gear train shown in Fig. 2 below, which consists of three gears, G1, G2, and G3, having radii R1, R2, and R3, and moments of inertia I1, I2, and I3 about their centers, respectively. G1 is connected to a bearing via a shaft having a torsional stiffness k such that, when G1 rotates by an angle \( \theta_1 \), an opposing torque \( k\theta_1 \) develops about its center. G2 is connected to a bearing which has an angular viscous coefficient c such that, when G2 rotates by an angle \( \theta_2 \), an opposing torque \( c\theta_2 \) develops about its center. G1 and G2 mesh without backlash (no slipping), and G2 and G3 are connected by a rigid, massless shaft. A driving counterclockwise torque \( T(t) \) is applied to G1.

a. Find the equation of motion of the system.

b. Find the equilibrium of the system, and linearize the equation of motion about it.

c. Reduce the system to the canonical form with equivalent mass, spring, dashpot, and generalized force. Identify the values of parameters \( \omega_n \) and \( \zeta \).

d. Determine the conditions on the values of \( I_1, I_2, I_3, k, \) and \( c \) for which the system is under-damped, critically damped and over-damped.

e. Determine the conditions on the values of \( I_1, I_2, I_3, k, \) and \( c \) for which the system is under-damped with a settling time of 1 s and a maximum overshoot of 10%.

f. For \( T(t) = T_0 \cos(\omega t) \), determine the particular solution to the ODE. Determine the value of the driving frequency at which the resonance occurs (when \( \zeta = 0 \)).

---

Figure 2. Gear Train
Problem 4. Mass-spring-dashpot in rotating frame

A mass $m$ lies in a frictionless groove, on a disk that lies on the horizontal plane and rotates at a constant angular velocity $\Omega$, as shown in Fig. 3 below. The mass is attached to the center of the disk by a spring and dashpot system, with constants $k$ and $c$ respectively.

a. Find the equation of motion of the system.
b. Find any equilibrium point(s).
c. Reduce the system to the canonical form with equivalent mass, spring, and dashpot. Identify the values of parameters $\omega_n$ and $\zeta$.
d. For what values of $\Omega$ are the equilibrium points unstable?
e. Assuming $\Omega$ is chosen so that the equilibrium is stable, determine the conditions on the values of $m$, $k$ and $c$ for which the system is under-damped, critically damped and over-damped.

Figure 3. Mass-spring-dashpot in rotating frame
Problem 5. Automobile suspension

A car's suspension system can be modeled as shown in the figure below. Assume the car is made of two equal masses $m$ attached via a spring of constant $k$ and a dashpot of constant $c$. The vertical position of the lower mass, $y_2(t)$, is a known function determined by the terrain on which the car rides. Note that gravity acts.

a. Find the equation of motion of the system.
b. Find the equilibrium position and examine whether it is stable. Linearize the equation of motion if necessary.
c. Determine the conditions on the values of $m$, $r$, $k$ and $c$ for which the system is under-damped, critically damped and over-damped.
d. Assume the vehicle travels horizontally at a constant speed over a sinusoidal terrain, such that $y_2(t) = h \sin(\Omega t)$. Determine the particular solution to the ODE. For what value of $\Omega$ will resonance occur (when $\zeta = 0$)?

Figure 4. Automobile suspension