1 Two carts

The two carts shown below roll on massless wheels on a horizontal surface. Derive the equations of motion for this system using Lagrange’s equations. Use $x_1$ and $x_2$ as generalized coordinates describing the position of masses $m_1$ and $m_2$ respectively, measured from the static equilibrium configuration in the absence of forces $F_1$ and $F_2$.

Put your equations of motion in the following matrix form:

$$M \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + C \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = F$$

where $M$, $C$ and $K$ are $2 \times 2$ matrices and $F$ is a $2 \times 1$ vector.
2 Centrifuge

A horizontal frictionless tube rotates about a vertical axis at a constant angular velocity $\Omega$. Two particles, $m_1$ and $m_2$, (connected by a spring), are placed inside the tube as shown. Derive equations of motion for this system. The unstretched length of the spring is $l_0$. 

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3 Rigid body pendulum

A pendulum consists of a rod of length $L$, mass $m$, and centroidal moment of inertia $\frac{1}{12}mL^2$ with a frictionless pivot at one end. The pendulum is suspended from a flywheel of radius $R$ and mass $M$ which can rotate about the fixed point $O$, as shown below.

i. Select a complete and independent set of generalized coordinates.

ii. Derive the Lagrangian equations of motion without making any approximations (small angles, etc.).