• “Closed book and notes”, one sheet of formulas allowed.

• Individual effort.

• Read all problems first.

• This quiz contains 6 printed pages.
Figure for Problem 1:
Problem 1: (50 Points)

A pendulum of mass $M$ with a massless rigid rod of length $L$ hanging from a fixed frictionless pin $O$ is initially at rest in a field with acceleration due to gravity $g$. The mass $M$ can be considered as a point mass (particle with negligible dimensions). A particle of mass $m$ and horizontal velocity $v$ flying at a vertical distance $d'$ from the pin collides with the pendulum rod and sticks to it.

a) a i. Explain why kinetic energy and linear momentum are not in general conserved during the collision.

a ii. Determine the angular velocity and kinetic energy of the system immediately after the collision, and the kinetic energy loss during the collision, as a fraction of the initial kinetic energy of the mass $m$.

a iii. Determine the impulse from the pin $O$ on the rod during the time of the collision.

**Hint:** Impulse is the integral of the force $\overrightarrow{F}(t)$ over the time of collision $\vec{J} = \int_{0}^{t} \overrightarrow{F}(t) \, dt$, assuming the time interval of collision is very small.

(2 + 10 + 5 = 17 Points)

b) Determine the second order (nonlinear) ordinary differential equation of motion of the system after the collision. The motion can be described in terms of the angle $\theta(t)$ the rod makes with the vertical line through $O$.

(10 Points)

c) c i. Determine the maximum angle $\theta_{\text{max}}$ the pendulum will make with the vertical after the collision.

c ii. Determine under what condition for the initial speed $v$ will the maximum angle of the pendulum after collision be equal to $\pi$ radians.

(5 + 3 = 8 Points)

d) Determine an expression for the force from the rod on the pin after the collision as a function of $\theta(t)$.

**Hint:** Eliminate any dependence on $\dot{\theta}(t)$ and leave only the dependence on $\theta(t)$ using an appropriate conservation law valid after the collision.

(10 Points)

e) Determine an integral expression for the time that will elapse between the collision and the maximum angle $\theta_{\text{max}}$ of question (c i).

**Hint:** Using an appropriate conservation law valid after the collision (just as in question d), express $\frac{d\theta}{dt} = f(\theta)$, where $f(\theta)$ is a function of $\theta$.

(5 Points)
Problem 2: (50 Points)

A ship is traveling at a constant velocity $V_{\text{mean}}$ with respect to the earth, along the axis $OX$, where $OXYZ$ is an inertial system (with unit vectors $\hat{i}, \hat{j}, \hat{k}$) attached to the earth. A moving system $Gxyz$ (with unit vectors $\hat{I}, \hat{J}, \hat{K}$) is attached to the ship at its center of gravity $G$, so that the axis $Gx$ makes an angle $\theta(t)$ to the horizontal axis $OX$. Due to head seas (a storm with waves whose crests are parallel to $OY$ axis and propagating along $OX$ axis) the ship oscillates in the vertical plane heaving and surging, and rotates about an axis orthogonal to its center plane (ie. pitching). The velocity of $G$ with respect to the inertial frame is

$$\overrightarrow{V_G} = V_{\text{mean}}\hat{i} + u(t)\hat{j} + w(t)\hat{k}$$

where $u(t)$ and $w(t)$ are the surge and heave velocities of the ship. The angular velocity of the ship with respect to $OXYZ$ is $\vec{\omega} = \omega_2(t)\hat{J}$, where $\omega_2(t) = \dot{\theta}(t)$, also known as pitch angular velocity.

At the same time the ship’s propeller of radius $R$ is rotating with its shaft at a constant angular velocity $\Omega$ with respect to the ship, ie.

$$\vec{\omega}_{\text{propeller, ship}} = \Omega\hat{i}$$

a) Determine the inertial velocity $\overrightarrow{v_p}$ and acceleration $\overrightarrow{a_p}$ of the center of the propeller, $P$ using the ship fixed system $Gxyz$ for your calculations. For simplicity of algebra express $\overrightarrow{v_p} = V_{\text{mean}}\hat{i} + \alpha\hat{j} + \beta\hat{k}$ and $\overrightarrow{a_p} = \gamma\hat{i} + \delta\hat{k}$, and determine $\alpha$, $\beta$, $\gamma$ and $\delta$. (15 Points)

b) Determine the angular velocity of the propeller with respect to $OXYZ$ and its time rate of change with respect to $OXYZ$. (5 Points)

c) The line $PT$ makes an angle $\phi(t)$ with the $Y$ direction (same as direction $y_1$) and assume, for simplicity that the line $PT$ is on a plane $Py_1z_1$ parallel to the plane $Gyz$. Note that $\dot{\phi}(t) = \Omega$. Find the inertial velocity $\overrightarrow{v_T}$ and the acceleration $\overrightarrow{a_T}$ of the tip $T$ for an arbitrary $\phi(t)$ and in particular when $\phi = \frac{3\pi}{2}$, ie. when the tip $T$ is at the lowest point. Hint: Use a system $Px_1y_1z_1$ fixed to the ship with origin at the propeller center $P$ and with the axes parallel to $Gxyz$, respectively. (20 Points)

d) Imagine that a mussel of mass $m$ happens to be attached to the tip $T$, of the propeller. For the condition of question (c) and $\phi = \frac{3\pi}{2}$ find the contact force from the blade on the mussel assuming the (static and dynamic) fluid force on the mussel is known and equal to $\overrightarrow{F_j(t)}$. Note that the acceleration of gravity is $g$. Assume that the propeller does not emerge from the ocean surface during the oscillations of the ship. (10 Points)