2.004 Dynamics and Control II
Spring 2008

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Reading:
- Nise: 4.1 – 4.8

1 The Time-Domain Response of Systems with Finite Zeros

Consider a system:

\[ G(s) = \frac{K(s + b)}{s^2 + 2s + 5}. \]

we have seen that we can consider this as two cascade blocks

Then if the response of the system \( 1/D(s) \) is \( v(t) \), then

\[ y(t) = \frac{dv}{dt} + bv(t) \]

and as the zero (at \( s = -b \)) moves deeper into the l.h. s-plane, the relative contribution of the derivative term decreases

and the system response tends toward a scaled version of the all pole response \( v(t) \).

In general, the presence of the derivative terms in the response means that:

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- The response is faster (shorter peak-time $T_P$ and rise-time $T_R$).
- Greater overshoot in the response (if any). A zero may cause overshoot in the response of an over-damped second-order system.

### Example 1

The following MATLAB step response compares the response for the under-damped system

$$G(s) = \frac{5}{s^2 + 2s + 5}$$

with similar unity-gain systems with zeros at $s = -1, -2, -3$:

$$G(s) = \frac{5(s + 1)}{s^2 + 2s + 5}, \quad G(s) = \frac{5/2(s + 2)}{s^2 + 2s + 5}, \quad G(s) = \frac{5/3(s + 3)}{s^2 + 2s + 5}$$

Note the increase in the overshoot, and the decrease in $T_P$ as the zero approaches the origin.
Example 2

The following MATLAB step response compares the response for the unity-gain overdamped system

\[ G(s) = \frac{12}{s^2 + 7s + 12} \]

with two real poles at \( s = -3 \) and \( s = -4 \) with the similar system with a zeros at \( s = -1 \):

\[ G(s) = \frac{12(s + 1)}{s^2 + 7s + 12} \]

Note the overshoot caused by the zero, but that the overshoot is not oscillatory. Clearly the rise-time \( T_R \) is much shorter for the system with the zero.

2 The Time-Domain Response of Systems where the Order of the NUMERATOR equals the Order of the Denominator

Consider systems of the form

\[ G(s) = \frac{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} \]
where the degree of the numerator equals that of the denominator. In such systems it is possible to do polynomial division and write the transfer function as
\[
G(s) = \frac{N(s)}{D(s)} = K + \frac{N'(s)}{D(s)}
\]
where \(N'(s)\) is a polynomial of degree less than that of \(D(s)\).

For example, a system with transfer function
\[
G(s) = \frac{s + a}{s + b}
\]
may be written
\[
G(s) = 1 + \frac{b - a}{s + a},
\]
which may be represented in block-diagram form

\[
\begin{align*}
U(s) &\rightarrow 1 \\
&\downarrow \quad b - a \quad \downarrow
\\
&\quad \downarrow s + a \\
&\rightarrow Y(s) + \rightarrow y(t)
\end{align*}
\]
showing a direct feed-through of the input into the output. In other words, when the order of the numerator is the same of the denominator the input will appear directly as a component of the output.

The step-response \(y_{step}(t)\) of this system will therefore be
\[
y_{step}(t) = u_s(t) + \frac{b - a}{a} \left(1 - e^{-at}\right)
\]
where \(u_s(t)\) is the unit-step (Heaviside) function.

Note:
- That \(y_{step}(0^+) = 1\), that is there is a step transient in the response (which does not occur if the order of \(N(s)\) is less than that of \(D(s)\)).
- The steady-state step response \(y_{ss} = b/a\), and if \(b > a\) then \(y_{ss} > 1\), while if \(a > b\) \(y_{ss} < 1\).

The following MATLAB plot shows the step responses for the two systems
\[
G(s) = \frac{s + 6}{s + 4} \quad (b > a), \quad \text{and} \quad G(s) = \frac{s + 2}{s + 4} \quad (a > b)
\]
with step responses
\[
y_{step}(t) = 1 + \frac{2}{4} \left(1 - e^{-4t}\right) \quad \text{and} \quad y_{step}(t) = 1 - \frac{2}{4} \left(1 - e^{-4t}\right)
\]
Example 3

Find the step response of the following electrical circuit:

![Circuit Diagram]

The transfer function is

\[
G(s) = \frac{V_o(s)}{V(s)} = \frac{s + 1/R_1C}{s + (R_1 + R_2)/(R_1R_2C)}
\]

and with the values shown

\[
G(s) = \frac{V_o(s)}{V(s)} = \frac{s + 100}{s + 150} = 1 - \frac{50}{s + 150}
\]

The step response is therefore

\[
y_{step} = 1 - \frac{50}{150} \left(1 - e^{-150t}\right) = \frac{2}{3} + \frac{1}{3} e^{-150t}
\]

which is plotted below:
Example 4

Find the step response of the following third-order system:

\[ G(s) = \frac{2s^3 + 17s^2 + 13s + 12}{s^3 + 7s^2 + 6s + 5} \]
\[ = 2 + \frac{3s^2 + s + 2}{s^3 + 7s^2 + 6s + 5} \]

showing a direct feed-through term of amplitude two. From Maple-Syrep, the step response is

\[ y_{step}(t) = 2.4 - 0.5307e^{-6.157t} + 0.1307e^{-0.4213t} \cos(0.7966t) - 0.2667e^{-0.4213t} \sin(0.7966t) \]

from which \( y_{step}(0^+) = 2 \), and \( y_{ss} = 2.4 \). The step response is plotted below.
Step Response

Time (sec) vs. Amplitude

Amplitude vs. Time (sec)