Today’s goals

- **Monday**
  - Proof that the frequency response as function of frequency $\omega$ is simply the value of the transfer function at $s=j\omega$
  - Bode plots: amplitude and phase of the frequency response on a log-log plot
  - Bode plots for elementary 1st order systems: derivative; integrator; zero; pole

- **Today**
  - Frequency response and Bode plots of underdamped 2nd order systems
  - Cascading sub-systems: rules for Bode plots of systems with multiple poles and zeros
Elementary Bode plots: 1\textsuperscript{st} order

Normalized and scaled
Bode plots for

a. \( G(s) = s; \)
b. \( G(s) = 1/s; \)
c. \( G(s) = (s + a); \)
d. \( G(s) = 1/(s + a) \)

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Bode plot for underdamped 2nd order system

\[ G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \]

\[ G(j\omega) = \frac{1}{(\omega_n^2 - \omega^2) + j2\zeta\omega_n\omega} \]

Note: the Bode magnitude at \( \omega = \omega_n \) is

\[ -20\log 2\zeta. \]

This can be used as correction to the asymptotic plot.
Cascading 1st order subsystems

\[ G(s) = \frac{K(s + 3)}{s(s + 1)(s + 2)} = \frac{3K}{2} \left( \frac{s}{3} + 1 \right) \]

Magnitude plot

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Cascading 1st order subsystems

\[ G(s) = \frac{K(s + 3)}{s(s + 1)(s + 2)} = \frac{\frac{3K}{2}}{s(s + 1)\left(\frac{s}{3} + 1\right)} \]

Phase plot

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Cascading 1st and 2nd order subsystems

\[ G(s) = \frac{K(s + 3)}{(s + 2)(s^2 + 2s + 25)} = \frac{3}{50} \frac{K(s^2 + 2s + 25 + 1)}{(s + 1)} \]

Magnitude plot

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Cascading 1\textsuperscript{st} and 2\textsuperscript{nd} order subsystems

\[ G(s) = \frac{K(s + 3)}{(s + 2)(s^2 + 2s + 25)} = \frac{3}{50} \frac{K}{\left(\frac{s}{3} + 1\right)} \left(\frac{s}{2} + 1\right) \left(\frac{s^2}{25} + \frac{2s}{25} + 1\right) \]

Phase plot

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