2.007 Design and Manufacturing I
Spring 2009

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2.007 – Design and Manufacturing I

Gears: Terminology, Geometry, Gear Trains, Strength

Presented by Dan Frey on 17 MAR 2009

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Today’s Agenda

• Distribute homework #3
• Gears
  – Applications
  – Types
  – Terminology / nomenclature
  – Congugate action
  – Involute curve
  – Analysis & design
Applications of Gears

Sobel, Dava, *Longitude*
Spur Gears

• Transmit motion between parallel shafts
• Teeth are parallel to the axis of rotation
• This is the simplest kind of gear we’ll consider and most of today is dedicated to them
Gear Terminology

Diametral pitch (teeth per inch)
# of teeth on a gear with a 1 inch pitch diameter

Other Types of Gears

- Helical
- Bevel
- Rack
- Worm

Images courtesy of OSHA.
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Image courtesy of perlmonger on Flickr.
Early Gears

Roman watermills at Barbegal
300AD

Application for powering textile machinery
18th century

Drawings of waterwheels and gears removed due to copyright restrictions.
Conjugate Action

Let's say $\omega_A$ is a known. How can we determine $\omega_B$?

Let's say $\omega_A$ is a constant with time. Can we synthesize a shape of body $B$ so that $\omega_B$ is also constant with time?
What is the pitch point?

What is the line of action?

What are the relationships among these?
Sliding and Rolling

What is the relationship to the pitch circles?

When one body is driving another, do the surfaces slide, roll, or both?

How could you determine this?
Rack Cutting

- A way to get the relative motion you want
- Pick one shape as you wish
- Enforce the motion you want
- Cut away everything that interferes
Involute Profile

• How it is constructed
  – Demo

• Properties
  – Conjugate action
  – Allows design of whole sets of compatible gears
  – Conjugate action not sensitive to center distance variations
More Gear Terminology

From Shigley and Mischke

Image removed due to copyright restrictions. Please see http://commons.wikimedia.org/wiki/File:Gear_words.png

This geometry is not an involute.
Pressure Line

- Where the teeth contact, the surface normal defines a pressure line
- The force transmitted acts along this line
- The pressure line always includes the point of tangency between the pitch circles
- With the involute gear profile, the pressure line is constant
Gear Terminology

“Line of action” & “pressure line” & “generating line” are all synonymous

Pressure Angle

- The pressure line acts at some angle to the tangent of the pitch circles
- This angle can be chosen by the designer
- It affects
  - Separation forces
  - Tooth shape

From Shigley and Mischke

Figure by MIT OpenCourseWare.
A pair of gears are mated. One is driven at a set torque, the other is regulated at a set speed. The gears are the ones circled. What is the ratio of the separation forces and the total force on the bearing?

1. $\ll 0.3$
2. About 0.3
3. About 0.5
4. $\gg 0.5$
Contact Ratio

contact ratio = length of arc of action / pitch = average number of teeth engaged
Interference

This portion of profile is not an involute

Interference is on flank of driver during approach

This portion of profile is not an involute

Figure by MIT OpenCourseWare.

From Shigley and Mischke
Backlash

Hub Material:
303 Stainless Steel

Coil type
Stainless Springs

P.D. (Ref.)

O.D.

Set Screw Supplied

Spot Drill

Pin Style

1/2

3/16

L

0.0005

0.0000

0.312 +0.000

-0.003

*Clamp Style

Courtesy of W. M. Berg, Inc. Used with permission.
Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

Spur Gears
24, 32, 48, and 64 Pitch 1/8" Bore AGMA Quality 4
Cold Rolled Steel and Brass 20° Pressure Angle

COURTESY OF W. M. BERG, INC. USED WITH PERMISSION.
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. No, this is not technically possible
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. OK, but it will be weak
4. No, this is not technically possible
Ways Gears Fail

Exceed endurance limit in bending

Exceed static yield stress in bending

“stripping”

Image courtesy of deltaMike at Flickr.

Exceed endurance limit in contact stress

“pitting”

Images removed due to copyright restrictions. Please see
http://materials.open.ac.uk/mem/mem_mf6.htm
http://www.hghouston.com/x/39_gearpit.html
Stress in Gears

Image removed due to copyright restrictions. Please see p. 1 in
A Beam in Bending

\[ \sigma = \frac{M}{I/c} = \frac{6W_t l}{Ft^2} \]
Concept Question

- In selecting a gear of one inch pitch diameter, we are choosing between 48 and 24 pitch gear teeth. The effect on torque that can be transmitted before bending failure of the teeth is

1. Around a factor of 10
2. Around a factor of 4
3. Around a factor of 2
4. Less than a factor of 2
Strength of Gears

- Any good catalog will have a formula and tables
- What factors must enter the equation?
- Where do the teeth wear the most?
The Lewis Formula

\[ \sigma = \frac{W_t P}{F_y} \]

- **Diametral pitch (teeth/inch)**
- **Face width**
- **“Lewis form factor”**
- **Low form factor \rightarrow High stress**

Point of max stress due to bending

Figure by MIT OpenCourseWare.
Or Use a Canned Tool

Please see “Spur Gear Tooth Strength” at http://www.wmberg.com/tools/
Discussion Questions

- I glued the third stage teeth of this servo together
- Now I will apply a load to the output shaft (up to 10lbs)
- What’s going to happen?

Epoxy applied liberally here
Concept Question

• For a gear to provide the highest strength at a fixed diameter, we prefer

1. High pressure angle
2. Low pressure angle
3. It doesn’t matter much

Figure by MIT OpenCourseWare.
Contact Stress
(Hertzian Stress)
Contact Stress
Quantitative Characterization

\[
b(d_1, d_2) := \sqrt{\frac{2 \cdot F}{\pi \cdot 1} \cdot \frac{\left(1 - \nu_1^2\right)}{E_1} + \frac{1 - \nu_2^2}{E_2}}
\]

\[
p_{\text{max}}(d_1, d_2) := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2) \cdot 1}
\]
Simple Gear Trains

• A “simple” gear train has only one gear on each shaft
• How does this arrangement behave?
Compound Gear Trains

- A “compound” gear train has at least one shaft with multiple gears
- How does this arrangement behave?
Manual Transmissions

If you find just two axles in a machine, does that mean there are just two stages?

Image from Wikimedia Commons, http://commons.wikimedia.org.
Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-025.gif
Discussion Questions

• Are there any disadvantage to a helical gear as compared to a spur gear?
• How can the disadvantages be remedied?
• Is a helical gear set stronger than a spur gear of the same diameter, pitch, face width, & material?

Image from Wikimedia Commons, http://commons.wikimedia.org
Concept Question

A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?

1. Pitch rises, face width rises
2. Pitch rises, face width falls
3. Pitch falls, face width rises
4. Pitch falls, face width falls
Discussion Questions

• How many stages in this device?
• How do you suppose this number is chosen?
• Are the reduction ratios typically all nearly the same in all successive stages?
Differentials

- Allows shafts to move at different speeds
- Applies same torque to both
- Slippage problem

Image removed due to copyright restrictions. Please see [http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif](http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif)
Next Steps

• Begin Homework #3
• Next lecture Thursday 19 March
  – CAD case study
• Spring break
• Lecture Tuesday 31 March
  – More gears, and also springs
• HW#3 due 7 April
• Quiz #2 on 16 April
• Impounding week 29 April to 1 May
Planetary Gear Trains

• One or more of the gear axes are allowed to rotate
• aka “epicyclic”
• Used in
  – Power tools
  – Automatic transmissions
  – Gear boxes

Please also see
http://commons.wikimedia.org/wiki/File:Epicyclic_carrier_locked.png
Analysis of Planetary Gear Trains

Figure by MIT OpenCourseWare.
Name That Gear

What type of worm gear set is this?

1) Single-enveloping, single threaded
2) Single-enveloping, multi-threaded
3) Double enveloping single threaded worm gear
4) Double enveloping multi-threaded
What is the reduction ratio of this gear set?

1) 10:1
2) 20:1
3) 40:1
4) 80:1