Gears: Terminology, Geometry, Gear Trains, Strength

Presented by Dan Frey on 17 MAR 2009

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Today’s Agenda

• Distribute homework #3
• Gears
  – Applications
  – Types
  – Terminology / nomenclature
  – Congugate action
  – Involute curve
  – Analysis & design
Applications of Gears

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http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif
http://www.nmm.ac.uk/collections/displayRepro.cfm?reproID=A6263

Sobel, Dava, Longitude
Spur Gears

• Transmit motion between parallel shafts
• Teeth are parallel to the axis of rotation
• This is the simplest kind of gear we’ll consider and most of today is dedicated to them
Gear Terminology

Diametral pitch (teeth per inch) # of teeth on a gear with a 1 inch pitch diameter

Easily confused

Other Types of Gears

- **Helical**
- **Bevel**
- **Rack**
- **Worm**

Images courtesy of [perlmonger](http://commons.wikimedia.org) on Flickr.

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Courtesy OSHA.
Early Gears

Roman watermills at Barbegal
300AD

Application for powering textile machinery
18th century

Drawings of waterwheels and gears removed due to copyright restrictions.
Conjugate Action

NOTE: As discussed in class, the rotation of the driven body B can be inferred. The key is that the two bodies stay in contact even after a small increment of rotation. For this reason, the “gear ratio” is a function of the orientation of the line normal to the point of contact.

Let’s say \( \omega_A \) is a known. How can we determine \( \omega_B \)?

Let’s say \( \omega_A \) is a constant with time. Can we synthesize a shape of body B so that \( \omega_B \) is also constant with time?

Yes, generally one can synthesize a shape B for any shape A to attain conjugate action. An analog of a “rack cutting” procedure would generally do it.
**Pitch Point**

**Line of action**, also called 'Pressure line'. The line along which the force between two meshing gear teeth is directed. It has the same direction as the force vector. In general, the line of action changes from moment to moment during the period of engagement of a pair of teeth. For involute gears, however, the tooth-to-tooth force is always directed along the same line -- that is, the line of action is constant. This implies that for involute gears the path of contact is also a straight line, coincident with the line of action -- as is indeed the case.

**Pitch point** (p). The point where the line of action crosses a line joining the two gear axes.

http://en.wikipedia.org/wiki/Gear
What is the relationship to the pitch circles?

When one body is driving another, do the surfaces slide, roll, or both?

But at the pitch point where the pitch circles touch, there is purely rolling.

Generally both rolling and sliding are going on at the same time.

How could you determine this?

Draw the velocity vectors at the pitch point.
Rack Cutting

- A way to get the relative motion you want
- Pick one shape as you wish
- Enforce the motion you want
- Cut away everything that interferes

Figure by MIT OpenCourseWare.
Involute Profile

• How it is constructed
  – Demo

• Properties
  – Conjugate action
  – Allows design of whole sets of compatible gears
  – Conjugate action not sensitive to center distance variations
This geometry is not an involute.

Image removed due to copyright restrictions. Please see http://commons.wikimedia.org/wiki/File:Gear_words.png

From
Shigley and Mischke
Pressure Line

- Where the teeth contact, the surface normal defines a pressure line
- The force transmitted acts along this line
- The pressure line always includes the point of tangency between the pitch circles
- With the involute gear profile, the pressure line is constant

From Shigley and Mischke
“Line of action” & “pressure line” & “generating line” are all synonymous

Pressure Angle

- The pressure line acts at some angle to the tangent of the pitch circles
- This angle can be chosen by the designer
- It affects
  - Separation forces
  - Tooth shape

From Shigley and Mischke

Figure by MIT OpenCourseWare.
A pair of gears are mated. One is driven at a set torque, the other is regulated at a set speed. The gears are the ones circled. What is the ratio of the separation forces and the total force on the bearing?

1. << 0.3
2. About 0.3
3. About 0.5
4. >> 0.5

Answer = 1: The key thing is pressure angle which is 20 deg. The ratio of separation and total force is \( \sin 20 \text{ deg} \).
Contact Ratio

Contact ratio = length of arc of action / pitch = average number of teeth engaged
Interference

Interference is on flank of driver during approach.

This portion of profile is not an involute.

This portion of profile is not an involute.

Pressure line.

Figure by MIT OpenCourseWare.

From Shigley and Mischke
Backlash

Hub Material:
303 Stainless Steel

Coil type Stainless Springs

P.D. (Ref.)

O.D.

Set Screw Supplied

Spot Drill

Pin Style

L

3/16

1/2

1/8

.2498

+.0005

-.0000

.312

+.000

-.003

*Clamp Style

Courtesy of W. M. Berg, Inc. Used with permission.
Gear Selection

- Pitch
- Face width
- Material
- Pressure angle
- # of teeth
- Hub style, bore, etc.

Courtesy of W. M. Berg, Inc. Used with permission.
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of 0.32 inches. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. No, this is not technically possible

I’d say “3”. A 48 pitch gear of 1 inch pitch dia has 48 teeth. The requested gear has 0.32*48=15.4 teeth. Integers are better for the number of teeth.
You call up the number 1-800-232-BERG and ask that, for a special application, you want a 48 pitch spur gear, but with a pitch dia of half the smallest one in the catalog. They will probably say:

1. OK, no problem
2. OK, but it will cost a lot
3. OK, but it will be weak
4. No, this is not technically possible

I'd say both “2” and “3”. A 48 pitch gear is listed with 14 teeth. Half the dia will give a pinion with 7 teeth. It will be hard to make it and tricky to avoid lots of undercut.
Ways Gears Fail

Exceed endurance limit in bending

Exceed static yield stress in bending

“stripping”

Exceed endurance limit in contact stress

“pitting”

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http://materials.open.ac.uk/mem/mem_mf6.htm
http://www.hghouston.com/x/39_gearpit.html

Image courtesy of deltaMike at Flickr.
Stress in Gears

Image removed due to copyright restrictions. Please see p. 1 in
A Beam in Bending

\[ \sigma = \frac{M}{I/c} = \frac{6W_l}{Ft^2} \]

Figure by MIT OpenCourseWare.
Concept Question

• In selecting a gear of one inch pitch diameter, we are choosing between 48 and 24 pitch gear teeth. The effect on torque that can be transmitted before bending failure of the teeth is

1. Around a factor of 10
2. Around a factor of 4
3. Around a factor of 2
4. Less than a factor of 2

Answ= 3: Model the gear as a beam. Its depth goes down by a factor of 2 which by itself raises stress by a factor of 4. But the length of the beam also drops by a factor of 2 and that reduces stress linearly.
Strength of Gears

- Any good catalog will have a formula and tables.
- What factors must enter the equation?
  - 
  - 
  - 
- Where do the teeth wear the most?

Gear wear the most where they experience sliding motion the most. That’s away from the pitch circle.
The Lewis Formula

\[ \sigma = \frac{W_tP}{F_y} \]

- Diametral pitch (teeth/inch)
- Face width
- "Lewis form factor"
- Low form factor → High stress
- Point of max stress due to bending

Figure by MIT OpenCourseWare.
Or Use a Canned Tool

Please see “Spur Gear Tooth Strength” at http://www.wmberg.com/tools/
Discussion Questions

- I glued the third stage teeth of this servo together
- Now I will apply a load to the output shaft (up to 10lbs)
- What’s going to happen?

The gears nearest the output shaft will fail since they experience the highest loads. The mode for Delrin gears is most probably in bending or “stripping”. I would estimate that the smaller gear (black in the photo) will fail rather than the white one since it has a narrower base and so a lower Lewis factor.
Concept Question

• For a gear to provide the highest strength at a fixed diameter, we prefer

1. High pressure angle
2. Low pressure angle
3. It doesn’t matter much

Answ = 1. High pressure angle will help raise the Lewis factor and lower stress. Comparing 14.5 deg and 20 deg pressure angle (back a couple slides), the difference is 10 to 20%, so not a negligible difference.

Figure by MIT OpenCourseWare.
Contact Stress
(Hertzian Stress)
Contact Stress
Quantitative Characterization

\[
\begin{align*}
\sigma_x(z, d_1, d_2) & := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2)} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right) \\
\sigma_y(z, d_1, d_2) & := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2)} \left( \frac{1}{d_1} + \frac{1}{d_2} \right) \\
\sigma_z(z, d_1, d_2) & := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2)} \\
\tau_{yz}(z, d_1, d_2) & := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2)} \\
\tau_{xz}(z, d_1, d_2) & := \frac{2 \cdot F}{\pi \cdot b(d_1, d_2)} \\
\end{align*}
\]
Simple Gear Trains

• A “simple” gear train has only one gear on each shaft

• How does this arrangement behave?

The gears turn in alternating directions. Such an arrangement might be useful to get a motion the same direction as that of the servo but at a different rate. The speed of the driven (yellow) one is not a function of the dia of the middle (pink) one.
Compound Gear Trains

• A “compound” gear train has at least one shaft with multiple gears

• How does this arrangement behave?

The key thing is that the total reduction ratio is a product of the ratios of the two mating pairs.
If you find just two axles in a machine, does that mean there are just two stages?

No, you probably have a “reverting” arrangement with many compound gears sharing a single pair of shafts.
Discussion Questions

• Are there any disadvantage to a helical gear as compared to a spur gear?
• How can the disadvantages be remedied?
• Is a helical gear set stronger than a spur gear of the same diameter, pitch, face width, & material?

Yes, helical gears cause axial loads. They are also more expensive. You can pair up helical gears to cancel the axial loads resulting in a “herring-bone” pattern. Helical gears are not really stronger, they are mostly quieter and smoother.
Concept Question

A compound gear train is formed of eight gears. As we proceed from the pinion on the electric motor to the gear on the output shaft, how do the pitch and face width vary?

1. Pitch rises, face width rises
2. Pitch rises, face width falls
3. Pitch falls, face width rises
4. Pitch falls, face width falls
Discussion Questions

- How many stages in this device? 4 stages
- How do you suppose this number is chosen? Usually not much more than 10 to 1 ratios per stage are practical.
- Are the reduction ratios typically all nearly the same in all successive stages? No, less reduction ratio as we approach the output shaft.
Differentials

• Allows shafts to move at different speeds
• Applies same torque to both
• Slippage problem

Image removed due to copyright restrictions. Please see http://mossmotors.com/Graphics/Products/Schematics/SPM-027.gif
Next Steps

• Begin Homework #3
• Next lecture Thursday 19 March
  – CAD case study
• Spring break
• Lecture Tuesday 31 March
  – More gears, and also springs
• HW#3 due 7 April
• Quiz #2 on 16 April
• Impounding week 29 April to 1 May
Planetary Gear Trains

• One or more of the gear axes are allowed to rotate
• aka “epicyclic”
• Used in
  – Power tools
  – Automatic transmissions
  – Gear boxes

Please also see
http://commons.wikimedia.org/wiki/File:Epicyclic_carrier_locked.png

Courtesy NASA.
Analysis of Planetary Gear Trains

Figure by MIT OpenCourseWare.
Name That Gear

What type of worm gear set is this?

1) Single-enveloping, single threaded
2) Single-enveloping, multi-threaded
3) Double enveloping single threaded worm gear
4) Double enveloping multi-threaded
Follow up

What is the reduction ratio of this gear set?
1) 10:1
2) 20:1
3) 40:1
4) 80:1

40 teeth