FUNdaMENTALs for Design Analysis: Fluid Effects & Forces

Prof. A. H. Techet
2.00a/16.00a Lecture 4
Spring 2009
Water & Air

- **Hydrodynamics v. Aerodynamics**
  - *Water is almost 1000 times denser than air!*

- **Air**
  - Density
    \[ \rho = 1.2 \text{ kg/m}^3 \]
  - Dynamic Viscosity
    \[ \mu = 1.82 \times 10^{-5} \text{ N} \cdot \text{s/m}^2 \]
  - Kinematic Viscosity
    \[ \nu = \frac{\mu}{\rho} = 1.51 \times 10^{-5} \text{ m}^2/\text{s} \]

- **Water**
  - Density
    \[ \rho = 1025 \text{ kg/m}^3 \text{ (seawater)} \]
    \[ \rho = 1000 \text{ kg/m}^3 \text{ (freshwater)} \]
  - Dynamic Viscosity
    \[ \mu = 1.0 \times 10^{-3} \text{ N} \cdot \text{s/m}^2 \]
  - Kinematic Viscosity
    \[ \nu = 1 \times 10^{-6} \text{ m}^2/\text{s} \]

Fluid Properties @20°C
Hydrostatic Pressure
Pressure under water

• Pressure is a Force per Area \( (P = \frac{F}{A}) \)

Pressure is a Normal Stress

Pressure is isotropic.

How does it act on these 2D shapes?
Pressure increases with depth

• Hydrostatic Pressure

\[ \frac{dp}{dz} = -\rho g \]

• Pressure on a vertical wall:

Absolute Pressure
\[ p - p_a = -\rho g (z - h) \]

\[ \rho gh + p_a \]

Gauge Pressure
\[ p_g = \rho g (h - z) \]

\[ \rho gh \]

The NET pressure force acts at the CENTER of PRESSURE

For more details with the derivation of the hydrostatic equation, see the reading on pressure posted on the class webpage.
Pressure on a sphere at depth?

Pressure acts normal to the surface. By convention pressure is positive in compression. The total force is the integration of the ambient pressure over the surface area of the sphere.

\[ F = \int \int_S p \cdot \hat{n} \, dS \]
Archimedes’ Principle

Weight of the displaced volume of fluid is equal to the hydrostatic pressure acting on the bottom of the vessel integrated over the area.

\[
F_z = pA = (\rho g D) \times (L W) = \rho g \times (L D W) = \rho g \times \text{Volume}
\]
Center of Buoyancy

*Center of buoyancy is the point at which the buoyancy force acts on the body and is equivalently the geometric center of the submerged portion of the hull.*

To calculate the center of buoyancy, it is first necessary to find the center of area!

1) Calculate the Area of the body:

\[ A = \int y(x) \, dx \]

2) Find the 1\textsuperscript{st} moment of the area:

\[ M_{xx} = \int x(y) \, y \, dy \]
\[ M_{yy} = \int y(x) \, x \, dx \]

3) Calculate the coordinates for the Center of Buoyancy:

\[ x = \frac{M_{yy}}{A} \quad \text{and} \quad y = \frac{M_{xx}}{A} \]
Stability?

A *statically stable vessel with a positive righting arm.*

```
Heel Angle
θ
```

“self-righting”

```
Heel Angle
θ
```

*Statically unstable vessel with a negative righting arm.*
Fluids in Motion...
Fluids Follow Basic Laws

- Conservation of Mass
- Conservation of Momentum
- Conservation of Energy
Flows can be described simply

- **Streamlines** are lines *everywhere* parallel to the velocity *(no velocity exists perpendicular to a streamline)*
- **Streaklines** are instantaneous loci of *all* fluid particles that pass through point $x_o$
- **Pathlines** are lines that one single fluid particle follows in time

In *Steady* flow these are all the same! Steady flow does not change in time.
Conservation of Mass

What goes in must come out!

Control Volume: nozzle

\[ \begin{align*}
\text{Area:} & \quad A \\
\text{Area:} & \quad a
\end{align*} \]

\[ U \rightarrow V \]

\[ \text{Volume:} \quad \forall = A \cdot L \]

\[ \text{Length:} \quad L = U \cdot \Delta t \]

To conserve Mass:

\[ \begin{align*}
\text{Mass:} \quad m &= \rho \forall \\
\text{Volume:} \quad \forall &= A \cdot L \\
\text{Length:} \quad L &= U \cdot \Delta t \\
\therefore m &= \rho AU \cdot \Delta t
\end{align*} \]

\[ \begin{align*}
\rho AU \cdot \Delta t &= \rho aV \cdot \Delta t \\
\rho A \cdot \Delta t \left( \frac{m_{in}}{m_{in}} \right) &= \rho a \cdot \Delta t \left( \frac{m_{out}}{m_{out}} \right)
\end{align*} \]
Conservation of Momentum

Newton’s second law states that the time rate of change of momentum of a system of particles is equal to the sum of external forces acting on that body.

\[ \sum F_i = \frac{d}{dt} \{ mV \} \]

Forces:
- Gravity (hydrostatic)
- Pressure !!!
- Shear (viscous/friction)
- External Body

\[ \Sigma F_x = \rho dA - (\rho' + dP) dA = m \frac{dv}{dt} \]

\[ \Sigma F_x = -dP dA = m \frac{dv}{dt} \]
Pressure Along a Streamline

Bernoulli’s Equation
(neglecting hydrostatics)

\[ P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 = \text{Const} \]

General eqn. on streamline:

\[ P + \frac{1}{2} \rho v^2 = C \]

Atmospheric pressure

\[ P_{atm} = P_{\text{static}} \]

Stagnation point
(high pressure)

Streamline
(C = constant)

Stagnation pressure:

\[ P_o = P_{atm} + \frac{1}{2} \rho v^2 \]

NB: The body surface can also be treated as a streamline as there is no flow through the body.
Aero/Hydro-dynamic Forces

\[ L = \frac{1}{2} \rho V^2 \cdot C_L \cdot S \]
\[ D = \frac{1}{2} \rho V^2 \cdot C_D \cdot S \]
Aero/Hydro-dynamic Forces

\[ L = \frac{1}{2} \rho V^2 \cdot C_L \cdot S \]

\[ D = \frac{1}{2} \rho V^2 \cdot C_D \cdot S \]

Dynamic Pressure (like in Bernoulli’s equation!!)
Aero/Hydro-dynamic Forces

\[ L = \frac{1}{2} \rho V^2 \cdot C_L \cdot S \]
\[ D = \frac{1}{2} \rho V^2 \cdot C_D \cdot S \]

Empirical Force Coefficients
Aero/Hydro-dynamic Forces

\[ L = \frac{1}{2} \rho V^2 \cdot C_L \cdot S \]

\[ D = \frac{1}{2} \rho V^2 \cdot C_D \cdot S \]

Wing Planform Area
Aero/Hydro-foil Geometry

- **Leading Edge**
- **Trailing Edge**
- **Chord Line**
- **b**, span
- **t**, thickness
- **c**, chord
- **S** = Planform Area
  - **S** = \( b \times c \) for rectangular foil

**Aspect Ratio**:

\[
AR = \frac{b^2}{S}
\]
Lift on a hydrofoil

Flow around a hydrofoil (blade section)

© 1999, S.A. Kinnas

- Top pressure LOWER than bottom pressure
- Lift increases with $U^2$, Angle of Attack, Camber
- Top pressure DECREASES as Lift (or $U$) increases

Courtesy of Spyros A. Kinnas. Used with permission.
Coefficient of Lift for a NACA 0012 Airfoil as a function of angle of attack

NACA 0012

First two numbers indicate camber
Double zeros indicate symmetric foil
Coefficient of Lift for a NACA 0012 Airfoil as a function of angle of attack.

NACA 0012

Last two numbers indicate thickness of foil as % of total chord length.
Coefficient of Lift for a NACA 0012 Airfoil as a function of angle of attack

For a symmetrical foil:

\[ C_l = 2\pi \alpha \]  
\( (\alpha \text{ in radians!}) \)

\[ F_{\text{LIFT}} = \frac{1}{2} \rho U^2 C_l S \]

National Advisory Committee for Aeronautics (NACA)
Maneuvering with a Rudder

Images removed due to copyright restrictions.
Please see: Fig. 14-1 and 14-2 in Gillmer, Thomas Charles, and Bruce Johnson. Introduction to Naval Architecture. Annapolis, MD: U.S. Naval Institute Press, 1982.
Images removed due to copyright restrictions.
Please see Fig. 14-5, 14-6, and 14-9 in Gillmer, Thomas Charles, and Bruce Johnson. 
### Turning Moment on a Vehicle

#### Single Motor Turn

- Moment Arm, \( r \)
- Vehicle Forward speed

\[ M = r \times T \]

#### Turning a Vehicle with a Rudder

- Moment Arm, \( r \)
- Vehicle Forward speed

\[ M = r \times L \]

**Lift on the Rudder:**

\[ L = \frac{1}{2} \rho V^2 (2 \pi \alpha) A \]

**A = area of rudder \( \alpha \) in radians**

**Coefficient of Lift:**

\[ C_L = 2 \pi \alpha \]

---

**Sum of the Moments (Torques) about the CG equals the time rate of change of angular momentum**

\[ \Sigma M_{CG} = I \dot{\theta} \]

\( I = \text{moment of Inertia of the vehicle about CG}^* \)

*can calculate in Solidworks!"
Viscous Drag

Skin Friction Drag: $C_f$

Form Drag: $C_D$
due to pressure (turbulence, separation)

Streamlined bodies reduce separation, thus reduce form drag.
Bluff bodies have strong separation thus high form drag.
Friction Drag

The transfer of momentum between the fluid particles slows the flow down causing drag on the plate. This drag is referred to as friction drag.

Friction Drag Coefficient:

\[ C_f = \frac{F}{\frac{1}{2} \rho U^2 A_w} \]

(units [MLT⁻²])

(non-dimensional coefficient)

\[ A_w = \text{Wetted Area} \]

**THIS IS A SHEAR FORCE THAT COMES FROM SHEAR STRESS AT THE WALL!**
Flat Plate Friction Coefficient

Variation of drag coefficient with Reynolds number for a smooth flat plate parallel to the flow.

$F = \frac{1}{2} \rho U^2 C_f A_w$

$L = 10^5 10^6 10^7 10^8 10^9$

$0.001 0.002 0.004 0.006 0.008$

$0.001 0.002 0.004 0.006 0.008$

$10^5 2 5 10^6 2 5 10^7 2 5 10^8 2 5 10^9$

Figure by MIT OpenCourseWare.
Viscous flow around bluff bodies (like cylinders) tends to separate and form drag dominates over friction drag.
Drag on Bodies

Drag acts inline with velocity.

Image removed due to copyright restrictions. Please see http://www.onera.fr/photos-en/tunnel/images/220006.jpg
Form Drag

• Drag Force on the body due to viscous effects:

\[ F_D = \frac{1}{2} \rho U^2 C_D A \]

• Where \( C_D \) is found empirically through experimentation

• \( A \) is profile (frontal) area

• \( C_D \) is Reynolds number dependent and is quite different in laminar vs. turbulent flows

Form Drag or Separation Drag or Pressure Drag (same thing!)
Flow Separating from a Cylinder

- Low pressure
- High pressure
- Stagnation pressure
- Separation point
- 'Negative' velocities (separated flow)
- Wake

Figure by MIT OpenCourseWare.
Classical Vortex Shedding

Alternately shed opposite signed vortices

Image removed due to copyright restrictions.
Please see http://en.wikipedia.org/wiki/File:Viv2.jpg
Vortex shedding results from a wake instability

Images removed due to copyright restrictions.
Please see Fig. 25-32 in Homann, Fritz. "Einfluß großer Zähigkeit bei Strömung um Zylinder."
Forschung auf dem Gebiete des Ingenieurwesens 7 (January/February 1936): 1-10.
Reynolds Number?

Non-dimensional parameter that gives us a sense of the ratio of inertial forces to viscous forces

\[ \text{Re} = \frac{\text{(inertial forces)}}{\text{(viscous forces)}} = \frac{(\rho U^2 L^2)}{(\mu UL)} = \frac{\rho UL}{\mu} \]

\[ v_{\text{water}} = \frac{\rho}{\mu} = 10^{-6} \text{m}^2/\text{s} \]

L = 1-10 mm

L = 9.5 m

Speeds: around 2*L/second

Speeds: in excess of 56 km/hr

Courtesy NOAA.

Image from Wikimedia Commons, http://commons.wikimedia.org
Reynolds Number Dependence

Regime of Unseparated Flow

A Fixed Pair of Foppl Vortices in Wake

Two Regimes in which Vortex Street is Laminar

Transition Range to Turbulence in Vortex

Vortex Street is Fully Turbulent

Laminar Boundary Layer has Undergone Turbulent Transition and Wake is Narrower and Disorganized

Re-establishment of Turbulent Vortex Street

Regimes of fluid flow across smooth circular cylinders (Lienhard, 1966).

\[ \text{Re} = \frac{\text{(inertial forces)}}{\text{(viscous forces)}} = \frac{(\rho U^2 L^2)}{(\mu U L)} = \frac{\rho U L}{\mu} \]

- \( R_d < 5 \)
- \( 5 - 15 < R_d < 40 \)
- \( 40 < R_d < 150 \)
- \( 150 < R_d < 300 \)
- \( 300 < R_d < 3 \times 10^5 \)
- \( 3 \times 10^5 < R_d < 3.5 \times 10^6 \)
- \( 3.5 \times 10^6 < R_d \)

Transition to turbulence

Figure by MIT OpenCourseWare.
Drag Coefficient for a smooth circular cylinder as a function of Reynolds number.

\[ Re = \frac{VD}{V} \]

Figure by MIT OpenCourseWare.
Drag Coefficient: Sphere

Drag coefficient of a smooth sphere as a function of Reynolds number.

Theory due to Stokes

Figure by MIT OpenCourseWare.
Trade-off between Friction and Pressure drag

c = Body length inline with flow

\*t = Body thickness

Images removed due to copyright restrictions.
Please see Fig. 7.12 and 7.15 in White, Frank M. *Fluid Mechanics*. Boston, MA: McGraw-Hill, 2007.
2D Drag Coefficients

For 2D shapes: use $C_D$ to calculate force per unit length.

Use a “strip theory” type approach to determine total drag, assuming that the flow is uniform along the span of the body.

Images removed due to copyright restrictions.
Trade-off between Friction and Pressure drag

c = Body length inline with flow

\[ t = \text{Body thickness} \]

Images removed due to copyright restrictions.
3D Drag Coefficients

Images removed due to copyright restrictions.
More 3D Shapes

Images removed due to copyright restrictions.
Calculating Drag on a Simple Structure

Example: Spherical vehicle with motors

Vehicle Forward speed

$\text{Drag} = \frac{1}{2} \rho V^2 C_D A$

$A = \text{front area} = \pi r^2$

$C_D = \text{drag coefficient}$

(depends on velocity)

$C_D = 0.4-0.5$ (laminar)

or 0.2 (turbulent)

$\text{Drag}_{\text{motors}} \sim \frac{1}{2} \rho V^2 C_D A$

$A = \text{front area} = \pi r^2$

$C_D = \text{drag coefficient}$

(depends on aspect ratio)

Assuming $L/D = 2$

$C_D = 0.85$

**This neglects the propellers which add some drag to the vehicle**

Use linear superposition to find total drag on a complex shape
Fluid Forces

Force on a surface ship is a function of $X$?

$F = f(\rho, \mu, g, U, L)$

**Dimensional Analysis:**

1) “Output” variable (Force) is a function of $N-1$ “input” variables ($\rho, \mu, g, U, L$). Here $N=6$.
2) There are $M=3$ primary dimensions (units) for the variables listed above: [Mass, Length, Time]
3) We can determine $P=N-M$ non-dimensional groups ($P=3$).
4) How do we find these groups?
Fluid Forces

Force on a surface ship is a function of $X$??

1) Fluid properties: density ($\rho$) & viscosity ($\mu$)
2) Gravity ($g$)
3) Fluid (or body) velocity ($U$)
4) Body Geometry ($L$)

$$F = f(\rho, \mu, g, U, L)$$

**Dimensional Analysis:**

1) “Output” variable (Force) is a function of $N-1$ “input” variables ($\rho, \mu, g, U, L$). Here $N=6$.
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3) We can determine $P=N-M$ non-dimensional groups ($P=3$).
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Dimensional Analysis

\[ F = f(\rho, \mu, g, U, L) \]

<table>
<thead>
<tr>
<th></th>
<th>( F ) [kg m/s^2]</th>
<th>( \rho ) [kg/m^3]</th>
<th>( U ) [m/s]</th>
<th>( L ) [m]</th>
<th>( \mu ) [kg/m/s]</th>
<th>( g ) [m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass [M]</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Length [L]</td>
<td>1</td>
<td>-3</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Time [t]</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Step 1) Make all variables containing mass non-dimensional in mass by dividing through by density: \( F/\rho \) and \( \mu/\rho \)
Dimensional Analysis

\[ \frac{F}{\rho} = f(\frac{\mu}{\rho}, g, U, L) \]

<table>
<thead>
<tr>
<th>Dimension</th>
<th>F/\rho [m^4/s^2]</th>
<th>\rho/\rho [-]</th>
<th>U [m/s]</th>
<th>L [m]</th>
<th>\mu/\rho [m^2/s]</th>
<th>g [m/s^2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass [M]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Length [L]</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Time [t]</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
</tr>
</tbody>
</table>

Step 2) Rewrite Matrix deleting density column and mass row
Dimensional Analysis

\[ \frac{F}{\rho} = f\left(\frac{\mu}{\rho}, g, U, L\right) \]

Step 3) Non-dimensionalize all variables containing time, using velocity: \( \frac{F}{\rho U^2} \) and \( \frac{\mu}{\rho U} \) and \( \frac{g}{U^2} \)
Dimensional Analysis

\[
\frac{F}{\rho U^2} = f\left(\frac{\mu}{\rho U}, \frac{g}{U^2}, L\right)
\]

<table>
<thead>
<tr>
<th></th>
<th>(F/\rho U^2) ([\text{m}^2/\text{s}^0])</th>
<th>(U/U) ([-\sim])</th>
<th>(L) ([\text{m}])</th>
<th>(\mu/\rho U) ([\text{m}^1/\text{s}^0])</th>
<th>(g/U^2) ([\text{m}^1/\text{s}^0])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [L]</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Time [t]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 4) Rewrite Matrix
Dimensional Analysis

\[
\frac{F}{\rho U^2} = f\left(\frac{\mu}{\rho U}, \frac{g}{U^2}, L\right)
\]

<table>
<thead>
<tr>
<th></th>
<th>(\frac{F}{\rho U^2}) [(m^2/s^0)]</th>
<th>(L) [m]</th>
<th>(\frac{\mu}{\rho U}) [(m^1/s^0)]</th>
<th>(\frac{g}{U^2}) [(m^{-1}/s^0)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [L]</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Step 5) Non-dimensionalize all variables containing length: \(F/\rho U^2L^2\) and \(\mu/\rho UL\) and \(gL/U^2\)
Dimensional Analysis

\[
\frac{F}{\rho U^2 L^2} = f\left(\frac{\mu}{\rho U L}, \frac{gL}{U^2}\right)
\]

<table>
<thead>
<tr>
<th></th>
<th>(F/\rho U^2 L^2)</th>
<th>(L/L)</th>
<th>(\mu/\rho U L)</th>
<th>(gL/U^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [L]</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Step 5) Your equation is non-dimensional! Yea!

But does it make sense??
Classical non-dimensional parameters in fluids

\[ \frac{F}{\rho U^2 L^2} = f\left(\frac{\mu}{\rho UL}, \frac{gL}{U^2}\right) \]

\[ C_F = \frac{F}{\frac{1}{2} \rho U^2 L^2} \]

**Force Coefficient**
(can be found through experiments and is considered an “empirical” coefficient, \( L^2 \) is equivalent to Area of object)

\[ \text{Re} = \frac{\rho UL}{\mu} \]

**Reynolds Number**
(important for all forces in air or water)

\[ Fr = \frac{U^2}{gL} \]

**Froude Number**
(in fluids typically only important when near surface of ocean/water)