1. a) Wind is the driving force for most ocean waves.

\[ \mathbf{u}_w = \frac{2\mathbf{u}_f}{\mathbf{u}_f} \]

b) \[ \mathbf{u}_f = \frac{\mathbf{f}}{\mathbf{t}} \]

The fluid particle follows the local velocity at each point in space.

Lagrange says: \( \frac{d\mathbf{p}}{dt} \) is the velocity of a fluid particle.

Euler says: \( \mathbf{u} \) is the local x velocity at each point in space.

When we talk about "motion of the free surface," we think about the velocity of fluid elements on the free surface. When we talk about the potential function and the velocity of a wave (\( \mathbf{u} = \frac{\mathbf{f}}{\mathbf{t}} \)), we are describing the velocity of the entire flow field. At the surface, the motion of the fluid element \( \frac{d\mathbf{p}}{dt} \) must be the same as what we prescribe for \( \mathbf{u} \).

By understanding the Lagrangian and Euler points of view, we can know when to use one or the other or set them equal.

c) \[ \mathbf{u} = \mathbf{u}_w \left( \frac{\cosh{(kz+\mathbf{H})}}{\sinh{\mathbf{H}}} \right) \cos{(kx-wt)} \]

\[ = \frac{\cosh{(kz)}}{\sinh{\mathbf{H}}} \left( \cosh{(kz)} \mathbf{H} + \sinh{(kz)} \sinh{(\mathbf{H})} \right) \]

\[ = \left( \cosh{(kz)} + \sinh{(kz)} \right) \left( e^{x+z} + e^{-x-z} \right) = e^{x+z} \]

\[ e^{x+z} \approx 0 \text{ when } z > \frac{\lambda}{2} = \frac{\pi}{w} \]

\[ e^{-\tau} = 0.04 \]

The submarine is pushed back forth as the waves pass.
2. **linear if** \( \left( \frac{2a}{\lambda} < \frac{1}{7} \right) \) the wave height is less than one seventh of the wavelength

**shallow if** \( \left( \frac{H}{\lambda} < \frac{1}{2a} \right) \) \( \Rightarrow \quad \omega = \sqrt{gH} \quad k = \frac{\omega}{\sqrt{gH}} \)

**intermediate if** \( \left( \frac{1}{2a} < \frac{H}{\lambda} < \frac{1}{2} \right) \) \( \Rightarrow \quad \omega^2 = gk \tanh(kH) \quad k = \text{solve iteratively} \)

**deep if** \( \left( \frac{1}{2} < \frac{H}{\lambda} \right) \) \( \Rightarrow \quad \omega^2 = k \gamma \quad k = \frac{0.2}{\gamma} \)

<table>
<thead>
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<th>( \omega ) (rad/s)</th>
<th>( H ) (m)</th>
<th>solve dispersion relation for ( k ) (m)</th>
<th>( \frac{\lambda}{k} )</th>
<th>( a ) (m)</th>
<th>( \frac{2a}{\lambda} )</th>
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</table>

Using `Matlab`:

\[
k = \text{fzero}(e(k) (9.81) + k \cdot \tanh(k \cdot H) - \omega^2, 1)
\]
3. For a linear free-surface wave:

\[ \eta = a \cos(kx-\omega t) \]

\[ u = a \omega f(z) \cos(kx-\omega t) \]
\[ \omega = a \omega f(z) \sin(kx-\omega t) \]
\[ f(z) = \frac{\cosh(kh)}{\sinh(kh)} \]
\[ \frac{\omega}{a} = \frac{f(z)}{\sinh(kh)} \]

\[ E = (m_0 + mw) \cdot a \]
\[ \frac{\partial u}{\partial t} = a \omega^2 f(z) \sin(kx-\omega t) \]
\[ \frac{\partial \omega}{\partial t} = -a \omega^2 f(z) \cos(kx-\omega t) \]
\[ m_0 \approx \rho \pi R^2 \cdot \frac{g}{2} \cdot c_0 \approx m_0 \]

\[ F_x = -2\pi R^2 \cdot a \omega^2 f(z) \sin^2(kx-\omega t) \]
\[ F_z = 2\pi R^2 \cdot a \omega^2 f(z) \cos(kx-\omega t) \]

7. Time \( t = 0 \):

* The wave packets move at group speed, \( V_g \)

\[ V_g = \left( \frac{c}{k^2} \right) \left( 1 \pm \frac{kh}{\sinh(kh) \cosh(kh)} \right) \]

\[ \frac{V_g}{V_p} = \frac{1}{2} - (t^* - 2) \]

Check: if \( V_g = \frac{t}{2} V_p \), then \( t^* = 2t \).
Buoyancy

\[ F_B = A \cdot w \cdot x \]

- Buoyancy is a restoring force that opposes the ship's motion away from its equilibrium position. As the ship goes down, buoyancy pushes up.

Frondé-Krylov

\[ F_{Fk} = \text{Constant} \cdot \eta \]

- Frondé-Krylov is a driving force that tries to pull the ship from its equilibrium position. As a wave passes, if the boat were fixed at its equilibrium position and a crest passes, it is like the boat is in deeper water, so a force pushes it up towards the surface, similar (heuristically) to buoyancy.

The equation of motion for the boat is:

\[
A \dot{x} + B \ddot{x} + C x = F(t)
\]

Frondé-Krylov is a driving force.

Buoyancy is a restoring force.