1) Consider the following control volume around a propeller where we have defined a slipstream boundary, such that the streamline that separates the flow going through the propeller from the free stream is taken to be the CV boundary. Assume that the flow is essentially one dimensional and is parallel in the streamwise direction at the inlet and outlet; also that the pressure along the slipstream boundary is equal to the ambient pressure.

![Diagram of propeller control volume](image)

a) Determine the thrust, $T$, and velocity of the flow across the propeller, $V$, in terms of the inlet and outlet speeds, $V_i$ and $V_e$, and the area of the propeller, $A$, and the fluid density, $\rho$.

b) Determine the power required to drive the propeller to generate the above thrust.

c) Show that efficiency of the propeller is equal to the ratio of the inlet velocity to the velocity across the propeller.
Efficiency here is defined as the power required to drive the propeller divided by the rate of change of Kinetic Energy of the slipstream between the inlet and outlet:

\[ P_{KE} = \frac{1}{2} \dot{m} \left( V_i^2 - V_e^2 \right). \]

\[ \eta = \frac{P}{P_{KE}} = \frac{V_i}{V_p} \]

d) Is the efficiency less than or greater than one? Explain.

2) An autonomous underwater vehicle (AUV) shaped like a torpedo is being designed to operate in an area of the Atlantic Ocean with strong currents around \( U = 3 \) m/s. In order to determine the thrust needed for the vehicle a smaller scale model is tested in the propeller tunnel. The model is 10\(^{th}\) the size of the real AUV. The model is 75 centimeters long and diameter is 7.5 cm. The front of the torpedo is a hemisphere and the aft is tapered to the propeller.

The full scale AUV will need enough thrust to maintain position in a 3 m/s current.

In order to determine the thrust needed to overcome the current the model will be hooked up to a force balance and the drag on the hull measured at a certain towing speed (analogous to incoming current).

a) What speed should the model be tested at to determine the necessary thrust on the full-scale vehicle?

b) If the drag measured on the model at this design speed is 10N, then what is the maximum thrust needed to overcome the 3 m/s current for the full-scale AUV?

c) List the most important non-dimensional parameters.