An autonomous land robot carries its own energy in the form of a $E(t = 0) = 700\text{Wh}$ (Watt-hour) battery. The robot can stay stationary at a cost of 18W to run its processor and instrumentation, or it can move with the additional locomotion cost of $35U^3$ Watts, where $U$ is the speed in $m/s$. The battery has a nonlinear discharge curve, that penalizes high loads: $-dE/dt = P + 0.005P^2$, where $P$ is the total power load (taken in Watts), and $dE/dt$ is also in Watts.

1. What is the longest duration mission that we can run, if the mission ends when $E = 0$?

The longest duration mission is one where we don’t move. We simply have the hotel load eating away at the battery, drawing 19.6W; the mission time is $T = 35.7h$. Formulas are in the attached MATLAB code.

2. What is the longest distance mission that we can run? Give the distance, duration, and vehicle speed for this mission.

The battery energy loss rate is a static function of load, so we have to just pick the best speed and run until the battery is dead. My code takes a numerical approach, sweeping through a number of possible speeds and picking out the case with greatest distance. You could also do it directly, by writing one equation for the distance as a function of the speed, and then finding the maximum using l’Hopital’s rule (zero slope). Either way, we find that the greatest distance is about 52.5km, over about 24.3h at a speed of 0.60$m/s$.

3. What is the longest distance mission, if we alternate resting periods and moving at a specific speed, with about half of the time spent in each state? Give the distance, duration, and the speed when moving.

The key here is that half of the time, we have only the hotel load to satisfy, and otherwise we have hotel plus locomotion. In all, the battery energy discharges at the average of the two rates. But the distance traveled is only the moving speed times half of the mission duration. I get a distance of 32.7km, over about 24.6h at a speed of 0.74$m/s$.

4. Comment on how you would use your findings in the design of effective mission plans.

The longer the mission, the more important it is to understand the hotel load as distinct from locomotion. This is especially true for a vehicle that has a lot of power-hungry sensors or communication gear on board, but may not have much locomotion cost. In this example, the hotel load costs us 20km of range when we operate in the stop-go mode.
AUTONOMOUS VEHICLE MISSION DESIGN, WITH A SIMPLE BATTERY MODEL

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**Graph 1:**
- Battery Energy Loss Rate, Watts
- Load, Watts

**Graph 2:**
- Constant Speed Case
- Distance, km
- Duration, hours
- Load, W

**Graph 3:**
- Stop/Go Case
- Distance, km
- Duration, hours
- Average Load, W

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**Table:**

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<tr>
<th>Load, Watts</th>
<th>Battery Energy Loss Rate, Watts</th>
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</tbody>
</table>

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**Equations:**

1. \[ E_{\text{loss}} = kU^2 \]
2. \[ \text{Distance} = \frac{E}{P_{\text{avg}}} \]
3. \[ \text{Duration} = \frac{E}{P_{\text{avg}}} \]

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**Graph Legend:**
- Solid line: Distance, km
- Dashed line: Duration, hours
- Dotted line: Load, W
- Dashed-dotted line: Average Load, W
clear all;

E0 = 700*3600 ;   % battery capacity, Joules
hotelLoad = 18 ;    % Watts
Uvec = 0:.01:1.3 ;  % a vector of speeds we’ll look at
squareCoefficient = 0.005 ;
propLoadCoefficient = 35 ;

P = 0:1:100 ;
Edot = P + squareCoefficient*P.^2 ;
figure(1);clf;hold off;subplot(211);
plot(P,Edot,‘LineWidth’,2);
hold on;
plot(P,P,’--k’);
xlabel(‘Load, Watts’);
legend(‘Battery Energy Loss Rate, Watts’,2);
grid;

Edot = hotelLoad + squareCoefficient*hotelLoad^2 ;
duration1 = E0 / Edot ;
disp(sprintf(‘Max duration mission is %g hours, 0 km.’, duration1/3600));

i = 1 ;
for U = Uvec,
    propulsionLoad = propLoadCoefficient*U^3 ;
    totalLoad = hotelLoad + propulsionLoad ;

    % Problem 2
    Edot2(i) = totalLoad + squareCoefficient*totalLoad^2 ;
    duration2(i) = E0 / Edot2(i) ;
    distance2(i) = duration2(i) * U ;

    % Problem 3
    Edot3(i) = (hotelLoad + squareCoefficient*hotelLoad^2) / 2 + ... 
                (totalLoad + squareCoefficient*totalLoad^2) / 2 ;
    % time-averaged value of Edot
    duration3(i) = E0 / Edot3(i) ;
distance3(i) = duration3(i) * U / 2; \% note division by two
i = i + 1;
end;
figure(2);clf;hold off;
subplot(211);
plot(Uvec,distance2/1000,Uvec,duration2/3600, '--',Uvec,Edot2,':', ...
     'LineWidth',2);
axis('tight');
xlabel('U, m/s');
title('Constant Speed Case');
legend('Distance, km','Duration, hours','Load, W',2);

[junk,ind] = sort(distance2);
ix = ind(end);
hold on;
plot(Uvec(ix),distance2(ix)/1000,'ro');
disp(sprintf('Max constant-speed distance mission is %g hours, %g km', ...
              duration2(ix)/3600, distance2(ix)/1000));

subplot(212);
plot(Uvec,distance3/1000,Uvec,duration3/3600,'--',Uvec,Edot3,':', ...
     'LineWidth',2);
axis('tight');
xlabel('U when moving, m/s');
title('Stop/Go Case');
legend('Distance, km','Duration, hours','Average Load, W',2);

[junk,ind] = sort(distance3);
ix = ind(end);
hold on;
plot(Uvec(ix),distance3(ix)/1000,'ro');
disp(sprintf('Max stop/go distance mission is %g hours, %g km', ...
              duration3(ix)/3600, distance3(ix)/1000));

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