37 Nyquist Plot

Consider the attached images; here are a few notes. In the plant impulse response, the initial condition before the impulse is zero. The frequency scale in the transfer function magnitude plots is $10^{-3} - 10^4$ radians per second. In the plot of $P(s)$ loci, the paths taken approach the origin from $\pm 90^\circ$, and do not come close to the critical point at $-1 + 0j$, which is shown with an x. In the plot of $P(s)C(s)$ loci, the unit circle and some thirty-degree lines are shown with dots. Also, the two paths in this plot connect off the page in the right-half plane.

Answer the following questions by circling the correct answer.

1. The overshoot evident in the open-loop plant is about
   (a) $120\%$
   (b) there is no overshoot since this is not a step response
   (c) $70\%$
   (d) $40\%$

2. The natural frequency in the open-loop plant is about
   (a) one Hertz
   (b) one radian per second - To compute this, you need a whole cycle.
   (c) 1.2 radians per second
   (d) six radians per second

3. Based on the plant behavior, $P(s)$ probably has
   (a) no zeros and one pole
   (b) one zero and one pole
   (c) no zeros and two poles
   (d) one zero and two poles - This plant has a zero at $+1$ (yes, a right-half plane zero, also known as an unstable zero) and two poles at $-0.1 \pm j$. You can tell it has two complex, stable poles because of the ringing in this impulse response. You can tell it has a zero because the output instantaneously moves to a nonzero value during the impulse - this could only be caused by a differentiator.

4. Compare the abilities of the plant hooked up in a unity feedback loop (i.e., with $C(s) = 1$), and of the designed closed-loop system, to follow low-frequency commands:
   (a) The $P(s)C(s)$ case has a lot more magnitude above one radian per second, and so it has a better command-following
   (b) $P(s)$ is nice and flat at low frequencies, so it is better at command-following
(c) $\mathbf{P}(s)C(s)$ has increasing values at lower frequencies and this makes it better - Setting $C(s) = 1$ will achieve about 10% tracking accuracy at low frequencies. The designed $\mathbf{P}(s)C(s)$ has a pole at or near the origin and hence is an integrator; this gives us no tracking error in the steady state.

The plant is stable, but lightly damped and it has an unstable zero. As you can guess from a quick check with a root locus, this is a difficult control problem, intuitively because the plant always moves in the wrong direction first. A PID cannot stabilize this system! I ended up using the loopshaping method in MATLAB’s LTI design tool; this gave a third-order controller with two zeros.

(d) The peak in $\mathbf{P}(s)$ is not shared by the other plot and this makes $\mathbf{P}(s)$ better at command-following.

5. Is the unity feedback loop stable, based on the loci of $\mathbf{P}(s)$?

(a) No: The path encircles the critical point once in the clockwise direction and that is all it takes, because the poles of $\mathbf{P}(s)C(s)$ are in the left-half plane - Note that the unstable zero in the plant is immaterial by itself. Nyquist’s rule is that stability is achieved if and only if $p = ccw$, where $p$ is the number of unstable poles in $\mathbf{P}(s)C(s)$, and $ccw$ is the number of counter-clockwise encirclements of the critical point.

(b) Yes: The path encircles the critical point once in the counter-clockwise direction

(c) Yes: The path encircles the critical point once clockwise and this is matched by a plant zero in the right-half plane

(d) No: The path encircles the critical point twice whereas it should only circle it once.

6. The designed compensator creates a stable closed-loop system, as is seen in the step response plot. The gain and phase margins achieved are approximately:

(a) 0.2 upward gain margin, 1.2 downward gain margin, and $\pm 40^\circ$ phase margin

(b) $\mathbf{1.2}$ upward gain margin, infinite downward gain margin, and $\pm 30^\circ$ phase margin

(c) infinite upward gain margin, 1.2 downward gain margin, and $\pm 30^\circ$ phase margin

(d) 1.2 upward gain margin, infinite downward gain margin, and $\pm 60^\circ$ phase margin
Loci of Plant $P(s)$
Loci of Loop Transfer Function $P(s)C(s)$

- Imaginary axis: $\omega > 0$ and $\omega < 0$
- Real axis: $-1.5$ to $1.5$

Closed-loop Step Response

- Time axis: 0 to 10 seconds
- Output axis: -4 to 2