2.019 Design of Ocean Systems

Lecture 13

Mooring Dynamics (II)

March 28, 2011
Cable configuration:

\[ s = \frac{T_H}{\omega} \sinh \left( \frac{\omega}{T_H} x \right) \]

\[ z + h = \frac{T_H}{\omega} \left[ \cosh \left( \frac{\omega}{T_H} x \right) - 1 \right] \]

Tension along the cable:

\[ T = T_H + \omega h + \left( \omega + \rho g A \right) z \]

\[ T_z = \omega s \]
Catenary Solution — Key Results (without Elasticity)

• Minimum line length required (or suspended length for a given fairlead tension) for gravity anchor:

\[ l_{\text{min}} = h \left( \frac{2T_{\text{max}}}{wh} - 1 \right)^{\frac{1}{2}} \]

• Horizontal force for a given fairlead tension:

\[ T_H = T - wh \]

• Horizontal scope (length in plan view from fairlead to touchdown point):

\[ x = \frac{T_H}{w} \sinh^{-1} \left( \frac{wl_{\text{min}}}{T_H} \right) \]

• Vertical force at the fairlead:

\[ T_z = wl_{\text{min}} \]
Simple Examples

- Given \( T_{\text{max}} = T_{\text{br}} = 1510 \text{ KN} \), \( w = 828 \text{ N/m} \), \( h = 25\text{ m} \), then

\[
l_{\text{min}} = h \left( \frac{2T_{\text{max}}}{wh} - 1 \right)^{\frac{1}{2}} = 25 \times \sqrt{\frac{2 \times 1510 \times 10^3}{828 \times 25}} - 1 = 300.93 \text{ m}
\]

\[
T_H = T - wh = 1510 \times 10^3 - 828 \times 25 = 1489 \text{ KN}
\]

\[
T_z = wl_{\text{min}} = 828 \times 301 = 249 \text{ KN}
\]

\[
x = \frac{T_H}{w} \sinh^{-1} \left( \frac{wl_{\text{min}}}{T_H} \right) = \frac{1489 \times 10^3}{828} \sinh^{-1} \frac{828 \times 301}{1489 \times 10^3} = 300 \text{ m}
\]

- Given \( x = 270 \text{ m} \), \( w = 828 \text{ N/m} \), \( h = 25 \text{ m} \), then

\[
h = \frac{T_H}{w} \left[ \cosh \left( \frac{wx}{T_H} \right) - 1 \right] \quad \rightarrow \quad T_H = 1200 \text{ kN}
\]

\[
T = T_H + wh = 1221 \text{ kN}
\]

\[
l_{\text{min}} = h \left( \frac{2T}{wh} - 1 \right)^{\frac{1}{2}} = 271 \text{ m}
\]

\[
T_z = wl_{\text{min}} = 828 \times 271 = 224 \text{ KN}
\]
Cable Load-Excursion Relation

\[ X = l - l_s + x \]

\[ X = l - h \left( 1 + 2 \frac{T_H}{wh} \right)^{\frac{1}{2}} + \frac{T_H}{w} \cosh^{-1} \left( 1 + \frac{wh}{T_H} \right) \]

Restoring Coefficient:

\[ C_{11} = \frac{dT_H}{dX} = w \left[ \frac{-2}{(1+2 \frac{T_H}{wh})^{1/2}} + \cosh^{-1} \left( 1 + \frac{wh}{T_H} \right) \right]^{-1} \]
Simple Example

Given: A ship experiences a total mean drift force (in surge) of 50KN, wave frequency oscillation of amplitude \( \zeta_1 = 3 \) m and frequency \( 2\pi/10 \) rad/s, what is the total tension in the cable?

Steady tension: \[ T_{0H} = 50kN \]

Mean position: \[ X_0 = 92.5m \]

Restoring coefficient: \[ C_{11}(T_{0H}) \approx 10kN/m \]

\[
T_H = T_{0H} + T_H(\omega) = T_{0H} + \left[ -C_{11}\zeta_1 \cos(\omega t + \alpha) \right] = 50 - 30 \cos(\omega t + \alpha)
\]

\[ |T_H| = 80kN \]  

Plus effect due to slowly varying motion

\[ T' = T'_H + wh \]
Catenary Solution —— Key Results (with Elasticity)

• Horizontal force for a given fairlead tension $T$:

$$T_H - AE \sqrt{(\frac{T}{AE} + 1)^2 - \frac{2wh}{AE}} - AE$$

• Minimum line length required (or suspended length for a given fairlead tension) for gravity anchor:

$$l_{\text{min}} = \frac{1}{w} \sqrt{T^2 - T_H^2}$$

• Vertical force at the fairlead:

$$T_z = wl_{\text{min}}$$

• Horizontal scope (length in plan view from fairlead to touchdown point):

$$x = \frac{T_H}{w} \sinh^{-1} \left( \frac{wl_{\text{min}}}{T_H} \right) + \frac{T_H l_{\text{min}}}{AE}$$

$AE$: stiffness of the cable
Analysis of Spread Mooring System

• Mean position of the body is determined by balancing force/moment between those due to environments and mooring lines

• Iterative solver is usually applied

Total mooring line force/moment:

\[ F_{1M} = \sum_{i=1}^{n} T_{Hi} \cos \psi_i \]

\[ F_{2M} = \sum_{i=1}^{n} T_{Hi} \sin \psi_i \]

\[ F_{6M} = \sum_{i=1}^{n} T_{Hi} \left[ x_i \sin \psi_i - y_i \cos \psi_i \right] \]

Total mooring line restoring coefficients:

\[ C_{11} = \sum_{i=1}^{n} k_i \cos^2 \psi_i \]

\[ C_{22} = \sum_{i=1}^{n} k_i \sin^2 \psi_i \]

\[ C_{66} = \sum_{i=1}^{n} k_i \left( x_i \sin \psi_i - y_i \cos \psi_i \right)^2 \]

\[ C_{26} = C_{62} = \sum_{i=1}^{n} k_i \left( x_i \sin \psi_i - y_i \cos \psi_i \right) \sin \psi_i \]