2.019 Design of Ocean Systems

Lecture 7

Seakeeping (III)

February 25, 2011
A regular plane progressive incident wave in deep water travels along the x-direction:

\[ \eta_I(x, t) = a \cos(\omega t - kx) \]

\[ \Phi_I(x, y, z, t) = -\frac{ga}{\omega} e^{kz} \sin(\omega t - kx) \]

To find the wave force and motion of the barge in the vertical direction using long-wave and strip theory assumptions.
Heave Wave Excitation on a Barge (I)

\[ F_{E3} = F_{I3} + F_{D3} \]

Using the strip theory (which is valid for \( B/L \ll 1 \)), we have:

\[ F_{E3} = \int_{-L/2}^{L/2} f_{E3}(x)dx, \quad F_{I3} = \int_{-L/2}^{L/2} f_{I3}(x)dx, \quad F_{D3} = \int_{-L/2}^{L/2} f_{D3}(x)dx \]

Froude Krylov force component:

\[ f_{I3}(x) = -\int_{-B/2}^{B/2} P_I(x)n_zdy \]

\[ = \int_{-B/2}^{B/2} (-\rho \Phi_t(x, y, z = -D, t))dy \]

\[ = B \rho gae^{-kD} \cos(\omega t - kx) \]

\[ F_{E3} = \int_{-L/2}^{L/2} f_{E3}(x)dx \]

\[ = \int_{-L/2}^{L/2} B \rho gae^{-kD} \cos(\omega t - kx)dx \]

\[ = \rho gaeB \left(\frac{2}{k}\right) e^{-kD} \sin \frac{kL}{2} \cos \omega t \]

In the limit \( \omega \to 0 \):

\[ F_{E3} \to \rho gaeBL \cos \omega t = \rho g\eta(t)(BL) \]
Heave Wave Excitation on a Barge (II)

Long-wave assumption: wave motion is a flow slowly varying in space and time. The wave diffraction effect is approximated by the added mass effect.

\[
f_{D3}(x, t) = A_{33}^{2D}(x) \dot{V}(x, t)
\]

\[
V(x, t) = \Phi_{Iz}(x, z = -D/2, t) = -\frac{gak}{\omega} e^{-kD/2} \sin(\omega t - kx)
\]

\[
\dot{V}(x, t) = -gak e^{-kD/2} \cos(\omega t - kx)
\]

\[
f_{D3}(x, t) = -gak e^{-kD/2} A_{33}^{2D} \cos(\omega t - kx)
\]

\[
F_{D3}(t) = \int_{-L/2}^{L/2} f_{D3}(x, t) dx = -2gak e^{-kD/2} A_{33}^{2D} \sin \frac{kL}{2} \cos \omega t
\]

In the limit \(\omega \to 0\): \(F_{D3} \to 0\)
Heave Wave Excitation on a Barge (III)

\[ F_{E3}(t) = F_{I3} + F_{D3} = \left[ \rho g a B e^{-kD} - g A_{33}^{2D} k a e^{-kD/2} \right] \left( \frac{2}{k} \right) \sin \frac{kL}{2} \cos \omega t \]

- Froude Krylov
- Added mass effect

\[ A_{33}^{2D} = C_a \left[ \rho \frac{\pi}{2} (B/2)^2 \right], \quad C_a \sim 1.0 \]
Radiation Force

Added mass coefficient: \( A_{33} = \int_{-L/2}^{L/2} A_{33}^D(x) \, dx = L A_{33}^D \)

Wave damping coefficient: \( B_{33} \to 0 \) with long-wave assumption

Radiation force: \( F_{R3} = -A_{33} \ddot{\zeta}_3(t) = -L A_{33}^D \ddot{\zeta}_3(t) \)

Restoring Force

\( F_{S3} = -C_{33} \zeta_3(t) = -\rho g B L \zeta_3(t) \)

Equation of Motion

\((M + A_{33}) \dddot{\zeta}_3 + B_v \dot{\zeta}_3 + \rho g B L \zeta_3(t) = F_{E3}(t)\)

If \( B_v = 0 \), \( \zeta_3(t) = \ddot{\zeta}_3 \cos(\omega t) \)

\[
\ddot{\zeta}_3(\omega) = \frac{[\rho g a B e^{-kD} - g A_{33}^D kae^{-kD/2} \left( \frac{2}{k} \right) \sin \frac{kL}{2}]}{-\omega^2(M + A_{33}) + \rho g B L}
\]
In the limit $\omega \to 0$:

$$\bar{\zeta}_3 = \frac{\rho g B L a}{-\omega^2 (M + A_{33}) + \rho g B L} = a$$

$$\zeta_3(t) = a \cos \omega t = \eta(x = 0, t)$$

Barge responds to move like a fluid particle in the limit of very long wave.

**Natural Frequency**

$$-\omega_n^2 (M + A_{33}) + \rho g B L = 0$$

$$\omega_n = \left( \frac{\rho g B L}{M + A_{33}} \right)^{1/2} = \left( \frac{g B}{B D + A_{33}^2} \right)^{1/2} = \left( \frac{g}{D + C_a (\pi/8) B} \right)^{1/2}$$

Natural period: $T_n = \frac{2\pi}{\omega_n}$ $T_n$ increases with D and B.

For example, for $D=20m$, $B=60m$, we have $T_n = 13s$. 
Sample Results for Heave Motion

Draft: D = 12 m
Width: B = 40 m

Natural period: $T_n = 11.4$ s
$B_v = 8\%$ critical damping

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Pitch Motion and Wave Loads on a Barge

Wave Excitation: \( F_{E5}(t) = \int_{-L/2}^{L/2} -x f_{E3} dx \)

\[
= \left[ \rho g B a e^{-kD} - g k a e^{-kD/2} A_{33}^2 \right] \int_{-L/2}^{L/2} [-x \cos(\omega t - kx)] dx
\]

Added mass and wave damping: \( A_{55} = \int_{-L/2}^{L/2} x^2 A_{33}^2 \) \( dx \), \( B_{55} = 0 \) as \( \omega \to 0 \)

Radiation moment: \( F_{R5} = -A_{55} \ddot{\zeta}_5(t) - B_{55} \dot{\zeta}_5(t) \)

Hydrostatic restoring moment: \( F_{S5} = -C_{55} \zeta_5(t) = -[\rho g \nabla (Z_B - B_G) + \rho g \int_{A_{wp}} x^2 ds] \zeta_5(t) \)

Moment of inertia: \( I_{55} = \rho DB \int_{-L/2}^{L/2} x^2 dx \)

From the equation of motion for pitch, we can get pitch motion: \( \zeta_5(t) = \bar{\zeta}_5 \sin \omega t \)

\[
\frac{\bar{\zeta}_5}{a} = \ldots .
\]
Sample Results for Pitch Motion

Draft: D=12 m
Width: B=40 m

B_v = 8% critical damping

Images by MIT OpenCourseWare.
Deck elevation at bow:

\[ Z_D = \zeta_3(t) - (L/2)\zeta_5(t) + H \quad \text{where } H \text{ is deck height} \]

Bottom elevation at bow:

\[ Z_B = \zeta_3(t) - (L/2)\zeta_5(t) - D \quad \text{where } D \text{ is draft} \]

Wave elevation at bow:

\[ Z_w = \eta(x = L/2, t) = a \cos(\omega t - kL/2) \]

If \( Z_D < Z_w \), wave overtopping occurs;
If \( Z_B > Z_w \), ship bottom is out of water
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