Dynamics

Dynamics: kinematics and kinetics of particles, rigid bodies and continua

Kinematics: studies motion without its cause

Kinetics: relates forces and torques to motion

Foundations of Dynamics: Newton's laws (axioms)

I. The existence of inertial frame

A free particle stays fix or moves uniformly along a line

II. In an inertial frame, "F = ma"

III. Action & Reaction forces are equal & act in opposite directions

This class applies these laws to particles, systems of particles, rigid bodies, systems of rigid bodies

Two approaches: 1) Newton-Euler approach (vectorial) → reaction forces

2) Lagrangian-Hamiltonian approach (scalar) → equations of motion

Newton-Euler mechanics (Newtonian)

1. Dynamics of a particle

\[ \text{Velocity, } \mathbf{v}(t) = \frac{d \mathbf{r}(t)}{dt} \]

\[ \mathbf{a}(t) = \frac{d \mathbf{v}(t)}{dt} = \frac{d^2 \mathbf{r}(t)}{dt^2} \]

\[ x(t) : \text{position of particle} \]

\[ (x_1, x_2, x_3) \text{ is an inertial frame} \]

(a) Linear momentum principle

\[ \mathbf{P} = m \mathbf{v} \quad \text{linear momentum} \]
Newton's Second Law: $F = ma$

If $F = 0$ then $a = 0$, so the object is at rest or moving with a constant velocity. This is the principle of conservation of linear momentum.

(b) Angular momentum principle

Define: $H_0 = \mathbf{r}_0 \times \mathbf{p}$

In words, $H_0$ is the moment of the linear momentum w.r.t. $B$.

Also define: $M_0 = \mathbf{r}_0 \times \mathbf{F}$

(resultant torque w.r.t. $B$)

\[
\frac{d}{dt}(H_0) = \frac{d}{dt}(\mathbf{r}_0 \times \mathbf{p}) = \mathbf{r}_0 \times \mathbf{p} + \mathbf{r}_0 \times \frac{d}{dt} \mathbf{p} = (\dot{\mathbf{r}} - \dot{\mathbf{r}}_0) \times \mathbf{p} + \mathbf{r}_0 \times \mathbf{F} = -\dot{\mathbf{r}}_0 \times \mathbf{p} + M_0 \quad \text{because (F \parallel \mathbf{p})}
\]

\[
\frac{d}{dt}(H_0) + \dot{\mathbf{r}}_0 \times \mathbf{p} = M_0
\]

(*) $H_0 = M_0$ if $\dot{\mathbf{r}}_0 = 0$ or $\dot{\mathbf{r}}_0 \parallel \mathbf{p}$ and the second includes the first.

Therefore, if $M_0 = 0$ AND (*) holds, then $H_0 = \text{constant}$

Conservation of angular momentum.
(c) Work-Energy Principle

Define $W_{12} = \int_{r_1}^{r_2} F \cdot dr$

Work done by resultant force

$$dr = v \cdot dt$$

$$F = m \ddot{v}$$

$$\Rightarrow W_{12} = \int_{r_1}^{r_2} m \ddot{v} \cdot v \cdot dt$$

$$= \int_{r_1}^{r_2} \frac{d}{dt}(\frac{1}{2} m v^2) dt$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Define $T = \frac{1}{2} m v^2$ Kinetic Energy

$$\Rightarrow W_{12} = T_2 - T_1$$

(work by $F$ equals to change in kinetic energy)

Assume that particle moves in a Force field $E(x, y) = F(x, y)$, such that

$\int_{r_1}^{r_2} E \cdot dr$ is independent of the path

between $r_1$ & $r_2$

Then $E(x)$ is Called Conservative.

Consequence

$\oint_{\Gamma} E \cdot dr = 0$

$\Gamma$: closed curve

$F \circ \Gamma \cap (-\Gamma_2)$

$F(x)$ is Conservative

Eq. field of gravity is a Conservative fields
By potential theory, for a conservative $E(x)$, there exists $V(x)$ (the potential) such that $E = -\nabla V$ (-grad $V$)

\[ E = -\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \frac{\partial V}{\partial x_3} \]

E.g. gravitational field

\[ \mathbf{F} = mg \rightarrow V = mgy \]

E.g. spring force

\[ V = \frac{1}{2}kx^2 \]

\[ \Rightarrow V(1) = \int_{r_1}^{r_2} E \, dr = \int_{r_1}^{r_2} (-V) \, dr = V_1 - V_2 \]

\[ \Rightarrow T_2 - T_1 = V_1 - V_2 \]

\[ \Rightarrow T_1 + V_1 = T_2 + V_2 \]

\[ \Rightarrow E = T + V \] total mechanical energy is conserved in a potential force field

Example: point mass slides on cylinder under the effect of gravity.

What is the fly-off angle?

How does $\phi^*$ depend on $R$, $m$?