Problem 1

A piston with mass $M$ is placed in a right-circular cylinder filled with viscous incompressible fluid of viscosity $\mu$ and density $\rho$. A small gap, $t$, between the piston and the cylinder allows the fluid to leak past the cylinder. The surface area of the piston face is $A$. The piston slowly settles to the bottom of the cylinder under the influence of gravity. You may assume that the density of the piston is much larger than the density of the fluid and that the ambient pressure is $P_{\text{amb}}$. Hint: in choosing a coordinate system, realize that the thickness of the gap is much smaller than the diameter of the piston.

a) What is the force on the upper and lower surfaces of the piston?

b) What is the net force on the vertical surface of the piston?

c) What is the velocity of the piston?

d) If the fluid is ambient air, the piston mass is 0.5 kg, the gap is 2 microns, the length of the piston is 25 cm and the diameter is 10 cm, how long does it take for the piston to travel 20 cm? The dynamics viscosity of the air is $\mu_{\text{air}} = 1.8 \times 10^{-5}$ Pa-s.
Problem 2
In a hydrostatic bearing, viscous fluid is pumped into the gap between a moving part and the structure that supports it. These bearings are capable of supporting very large forces (this type of bearing is used in automobiles, compressors and in very large equipment). In the diagram above, fluid is pumped into a slot “A” at high pressure. The fluid then flows from slot A into the gap “B” between the parts.

a) Please model the flow in gap B and explain/show that the expected pressure distribution is as indicated in the diagram.

b) If this bearing is 35 cm wide, what is the discharge rate of oil per meter of length of bearing to support a 75 kN per meter load with a clearance space, t, of 0.50 mm? (Assume $\mu_{oil} = 0.1 \text{ Pa-s}$, $\rho_{oil} = 800 \text{ kg/m}^3$).

Problem 3
A viscous liquid flows between two concentric cylinders of radii $R_1$ and $R_2$ where $R_1 < R_2$. The inner cylinder is at rest and the outer moves parallel to the common axis at speed $U$. Assuming that the pressure at the inlet and outlet of the concentric pipe is atmospheric and neglecting edge effects, show that the steady state velocity profile in the concentric pipe is given by

$$v_z(r) = \frac{U}{\ln \left( \frac{R_2}{R_1} \right)} \ln \left( \frac{r}{R_1} \right)$$