Topics: Hydrostatics + Surface Tension Examples

Problem 1

Consider the frictionless piston-cylinder system to the right. The mass of the piston and cylinder are $M_p$ and $M_c$, the cross-sectional area of the piston is $A$, the length of the cylinder is $L_o$, and the piston and cylinder walls are very thin. The cylinder is filled with air (modeled as an ideal gas with negligible weight); when the cylinder is held horizontally in ambient pressure $P_o$ as shown, the air occupies its entire volume (i.e., $A L_o$). Gravitational acceleration is $g$. The temperature of air may be assumed to be constant throughout the problem.

a) The cylinder is submerged vertically in water (density $\rho$) and attains equilibrium at depth $h$ as shown in part a) of the figure below. The air and surrounding water are in thermal equilibrium at a temperature $T$ (constant with depth). The goal of part a) is to obtain an expression for the depth $h$ at which the piston-cylinder system is in equilibrium.

i. By considering a force balance on the entire piston-cylinder-air system, determine the length of the trapped air column $L$.

ii. Determine the pressure $P$ of the trapped air.

iii. By considering a force balance on the piston, find the piston depth $h$ as shown.

b) The cylinder now has a small opening of diameter $d$ as shown in part b) of the figure. A meniscus will form at the liquid-air interface; the goal of part b) is to obtain the maximum diameter $d$ of the opening for which the air-water interface is stable within the cylindrical hole as shown. Reminder: $\Delta P = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$, where $\Delta P$ is the pressure difference across the interface and $r_1$ and $r_2$ are the radii of curvature of the interface.

i. If the three phase contact angle is $\theta$ and the liquid-air surface tension is $\sigma$, what is the radius of curvature $r$ of the liquid-air interface and the pressure difference $\Delta P$ across the interface, both in terms of given variables?

ii. By comparing this $\Delta P$ to the difference in pressure between the liquid immediately above the cylinder and the trapped air (from part a), find the maximum diameter $d$ for which the air-water interface is stable within the cylindrical opening as shown.
Part a) \[ h \text{ (from part a)} \]

\[ \text{Atmospheric pressure at water surface} = P_o \]

\[ L \]

\[ L_o \]

Part b) \[ h \text{ (from part a)} \]

\[ \theta \]

\[ d \]