Lecture 5
Solution Method for Beam Deflection

**Problem 5-1:** Consider the clamped-clamped elastic beam loaded by a uniformly distributed line load $q$.

![Beam Diagram]

a) Formulate the boundary conditions.

b) Find the deflected shape of the beam using the direct integration method.

c) Find the maximum deflection magnitude and location.

d) Determine the location and magnitude of the maximum stress in the beam.

**Problem 5-2:** Calculate the second moment of inertia of the beam cross section for:

a) Solid rectangular cross section of width $b$ and height $h$.

b) Thin-walled square box section of width and height $b$.

c) Solid circular cross section of radius $r$.

**Problem 5-3:** In wood construction building codes the beam deflections cannot exceed $L/360$ where $L$ is the length of the beam. Where do you think this requirement comes from? Choose a typical beam example and state clearly the formulation and your assumptions on the boundary conditions and loading. Using the deflection criteria estimate the fracture strain of the plaster board which is nailed directly to the ceiling beams (joist) in single home construction.
**Problem 5-4:** Given a beam with a “T” section subjected to pure bending shown in Figure 1, calculate:

a) the location of the neutral axis

b) the second moment of inertia

c) Find the shear stress distribution in the “T” section.

![Figure 1]

**Problem 5-5: Continuity Condition**

Solve the problem of a simply-simply supported beam loaded by a point force acting at eh symmetry plane, but at a distance $a$ from the left support

![Diagram](image)

In the notes of lecture 5 the solution of this problem was outlined, but not completed,

(a) Complete the derivation by calculating all four integration constants

(b) Proof that all continuity conditions are satisfied at $x=a$

(c) Show that in the limiting case of $a=L/2$ the solution is identical to one that was derived in class and you were asked to memorize
**Problem 5-6:**

Another problem to test your knowledge on continuity conditions.

A system of two identical beams shown in the figure below is a statically determined problem. The beams are rigidly welded, so that the angle remains 90 degrees.

![Diagram of two identical beams](image)

Determine

(a) The distribution of bending moment and shear forces in both segments.

(b) Find the deflected shape of both beams and make sketch.

(c) Find the relation between the tip-- load and the vertical displacement of the tip. (Hint: assume the rotations to be very small.)

(Note: Do not solve this problem using the Castigliano’s theorem, which is much simpler, but has not been covered yet.)