Lecture 8
Energy Methods in Elasticity

Problem 8-1: Consider an elastic cantilever beam loaded at its tip.

a) Specify the boundary conditions.

b) Derive the load-tip displacement relation using four methods presented in class.

Method I Solving uncoupled problems
Method II Solving coupled problem (direct integration)
Method III Castigliano Theorem
Method IV Ritz Method

Problem 8-2: Consider a beam of length L and bending rigidity EI which is fully clamped on both ends shown in Figure 1. The beam is subjected to a point force P at the midspan. Solve the problem (find the expression between the load and the deflection under the load) using:

a) The direct integration of beam equation with the suitable boundary conditions (exact solution).

b) The Ritz Method (approximate solution).

c) Compare the results and calculate the relative error of the Ritz Method.
Problem 8-3: Use Castigliano’s Theorem to calculate the horizontal deflection at point D in Figure.