Recitation 9

Buckling of Sections
Consider the box column shown:

Find the critical buckling load.

Column can buckle
*globally* (Euler) or
*locally* (plate)

Global (Euler) Buckling

\[ P_c = \frac{\pi^2 EI}{L^2} \]

\[ I_x = 2\left(\frac{b_1 h^3}{12} + b_1 h \left(\frac{b_2}{2}\right)^2\right) + 2\frac{h b_2^3}{12} \]
\[ I_y = 2\left(\frac{b_2 h^3}{12} + b_2 h \left(\frac{b_1}{2}\right)^2\right) + 2\frac{h b_1^3}{12} \]

\[ I_x < I_y \rightarrow \text{Will buckle about } x\text{-axis} \]

So \[ P_c = \frac{\pi^2 E}{L^2} \left(\frac{b_1 h^3}{6} + \frac{b_1 b_2^2 h}{2} + \frac{b_2^3 h}{6}\right) \]

Local (Plate) Buckling

Treat each side as an individual plate.
* Assume uniform compression → stress in each plate is the same.

\[ \text{Plate (1): } \sigma_1 = \frac{P_1}{hb_1} \]

\[ \text{Plate (2): } \sigma_2 = \frac{P_2}{hb_2} \]

For a plate simply supported on the loaded edges:

\[ P_c = \frac{k_c \pi^2 D}{b} \quad \text{(where } k_c \text{ depends on BC on other sides and dimensions)} \]

\[ D = \frac{Eh^3}{12(1-\nu^2)} \]

So

\[ \sigma_{cr,1} = \frac{P_{c1}}{hb_1} = \frac{k_{c1} \pi^2 D}{hb_1^2} \]

and

\[ \sigma_{cr,2} = \frac{P_{c2}}{hb_2} = \frac{k_{c2} \pi^2 D}{hb_2^2} \]

All plates buckle at the same time, so

\[ \sigma_{cr,1} = \sigma_{cr,2} \]

\[ \frac{k_{c1} \pi^2 D}{hb_1^2} = \frac{k_{c2} \pi^2 D}{hb_2^2} \rightarrow k_{c2} = k_{c1} \left( \frac{b_2}{b_1} \right)^2 \]

Total load = 2P₁ + 2P₂

\[ \rightarrow P_{c,\text{tot}} = 2P_{c1} + 2P_{c2} = 2 \left( \frac{k_{c1} \pi^2 D}{b_1} + \frac{k_{c2} \pi^2 D}{b_2} \right) \]

\[ = 2\pi^2 D k_{c1} \left[ \frac{1}{b_1} + \left( \frac{b_2}{b_1} \right)^2 \cdot \frac{1}{b_2} \right] \]

\[ = \frac{2\pi^2 D k_{c1}}{b_1} \left( 1 + \frac{b_2}{b_1} \right) \]
But what is $k_{c1}$? 
- In general, the adjacent plates on the unloaded edges will cause a bending moment (somewhere between simply supported and fully clamped)

(Plot on page 16 gives $k_{c1}$ as function of $\frac{b_2}{b_1}$)

(assumes $L/b_1 > 5$)

**Example**

$h = 2$ mm
$b_1 = 100$ mm
$b_2 = 50$ mm

Find the length, $L$, that marks transition between global and local buckling.

\[
P_{c,\text{global}} = \frac{\pi^2 E}{L^2} \left( \frac{b_1 h^3}{6} + \frac{b_1 b_2^2 h}{2} + \frac{b_2^3 h}{6} \right)
\]

\[
= \frac{\pi^2 E}{L^2} \left( \frac{100(2)^3}{6} + \frac{100(50)^2(2)}{2} + \frac{50^3(2)}{6} \right) = \frac{\pi^2 E}{L^2} (291,800 \text{ mm}^4)
\]

\[
P_{c,\text{local}} = \frac{2\pi^2 D k_{c1}}{b_1} \left( 1 + \frac{b_2}{b_1} \right) \quad (\text{From plot: } k_{c1} \simeq 5.2)
\]

\[
= \frac{2\pi^2 E h^3 k_{c1}}{b_1(12)(1 - \nu^2)} \left( 1 + \frac{b_2}{b_1} \right)
\]

\[
= \frac{2\pi^2 E (2)^3(5.2)}{100(12)(1 - 0.3^2)} (1 + 0.5) = \pi^2 E (0.114 \text{ mm}^2)
\]

Let $P_{c,\text{global}} = P_{c,\text{local}}$ and solve for $L$:

\[
\frac{\pi^2 E}{L^2} (291,800) = \pi^2 E (0.114)
\]

$L \simeq 1600 \text{ mm} = 1.6 \text{ m}$