Date Issued: Wednesday 3 September, 2014

Date Due: Wednesday 10 September, 2014, 9:30AM (bring hard copy to lecture)

As described in the course policies document, this is one of 5 homeworks you will complete in this course. Each of these count as 6% of your total grade. Full credit can generally only be earned by showing your work. This often includes making clear and well-labeled plots.

1) (5 points) Make a graph of $y = e^t$ for $-1 \leq t \leq 1$ either by hand or using MATLAB®.

What is the value of the function $y$ at $t=0$?

What is the value of the function $y$ at $t=1$?

What is the slope $dy/dt$ at $t=0$?

What is the slope $dy/dt$ at $t=1$?

2) (15 points) For each of the differential equations and solutions below, demonstrate that the proposed solution satisfies the differential equation.

   a. $2ty' - 4y + 12 = 0$  Solution: $y(t) = 2t^2 + 3$
   b. $t^2y' - y(1 - t) = t^2$  Solution: $y(t) = te^{-t} + t$
   c. $6y - \frac{1}{3}y'' = 0$  Solution: $y(t) = t^3$

3) (6 points) For each of these differential equations, indicate whether it is linear in $y$.

   a. $dy/dt + \sin y = t$
   b. $y' = t^2(y-t)$
   c. $y' + e^t y = t^{10}$
4) (4 points) What linear differential equation \(\frac{dy}{dt} = a(t)y\) is satisfied by \(y(t) = e^{\cos(t)}\) ?

5) (10 points) All solutions of \(\frac{dy}{dt} = -y + 2\) approach as steady state where \(\frac{dy}{dt} = 0\) and \(y = y_\infty\). That value, \(y_\infty\) is a particular solution. What null solution \(y_\infty = Ce^{-t}\) combines with the particular solution to satisfy \(y(0) = 4\) ?

6) (10 points) Find the solution of \(\frac{dy}{dt} + 2y = 6\) where \(y(0) = 1\). What is \(y_\infty\)? Make its graph.

7) (10 points) Draw the function that solves \(y' = H(t-T)\) where \(y(0) = 2\). Note \(H(t)\) is the unit step function.

8) (20 points)
   a) Find the function that solves \(y' - y = \delta(t-2)\) where \(y(0) = 3\). Note \(\delta(t)\) is the delta function.
   b) Make a graph of the solution (by hand or with a computer). Comment on any features of the graph that are notable to you.

9) (20 points) A model aircraft is pointed straight down with its engines off. At time \(t=0\) sec, it has just begun descent from a vertical climb maneuver and it has, momentarily, zero airspeed, \(V(0s)=0\) m/s. Its mass is 1.2 kg and its weight causes acceleration. To determine the effect of aerodynamic drag as speed builds, consider the drag force is given by \(\frac{1}{2} \rho V^2 S C_D\) where its drag coefficient \(C_D = 0.02\) and the area \(S\) is 0.22 m\(^2\) and density of the air \(\rho\) is the typical value for sea level about 1.3 kg/ m\(^3\). At time \(t=5\) sec, it deploys speed brakes so its drag coefficient changes suddenly to \(C_D = 0.08\).
   a) Write a differential equation modeling the evolution of airspeed \(V(t)\) from \(t=0\) sec to \(t=5\) sec.
   b) Find the solution to the equation in (a) satisfying the initial condition \(V(0s)=0\) m/s and find the speed at to \(t=5\) sec.
   c) Write a differential equation modeling the evolution of velocity from \(t=5\) sec onward and choose a condition to define an “initial” value problem.
   d) Find the solution to the equation in (c) or else describe as many features of the solution as you can infer within a reasonable time allocation.
   e) Estimate the value of the time \(t\) when the aircraft gets to within 5% of the steady state speed after the time that speed brakes were deployed.