Problem 1 (10 points)

A computer program is used to perform a plane strain analysis. A total Lagrangian formulation is employed with the given elastic material law. Consider the 4-node element shown at time “t”. For this time:

a) Calculate the deformation gradient.

b) Calculate the second Piola-Kirchhoff stresses.

c) Give the equation to calculate the Cauchy stresses. Do not actually do the arithmetic.

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1/2
\end{bmatrix}
\]

Poisson’s ratio = 0

Rigid body rotation only from time “t-\Delta t” to time “t”
**Problem 2 (10 points)**

A two-dimensional steady-state planar fluid flow analysis solving the Navier-Stokes equations assuming incompressible, very low Reynolds number flow is to be performed. The governing equations in the principle of virtual velocities are:

\[
\begin{align*}
\int_V \rho_i \left( \frac{\partial \mathbf{v}_i}{\partial t} + \mathbf{v}_i \cdot \nabla \mathbf{v}_i \right) dV + \int_V \nabla \cdot \mathbf{v} dV &= \mathbf{R} \\
\int_V \rho dV &= \mathbf{0} \\
\tau_{ij} &= -p \delta_{ij} + \mu \left( \mathbf{v}_{i,j} + \mathbf{v}_{j,i} \right) = -p \delta_{ij} + 2 \mu \mathbf{e}_{ij}
\end{align*}
\]

Assume that the Reynolds number is so small that the first term (the convection term) on the left-hand side of the first equation can be neglected (Stokes flow is assumed).

The 9/3 velocity/pressure element is to be used for the solution.

a) Evaluate the finite element matrix entry corresponding to the nodal velocity shown.

b) Evaluate the finite element matrix entry .

In each case, (a) and (b), give all integrals but do not perform any integration.

\[
\begin{bmatrix}
K(1,1) & \cdots \\
\vdots & \ddots & \ddots \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{v}_1 \\
\mathbf{v}_2 \\
\mathbf{v}_3 \\
\vdots \\
p_0 \\
p_1 \\
p_2
\end{bmatrix} = \cdots
\]

where \( \mathbf{v}_j \) is the velocity at node \( i \) into direction \( j \).

c) State briefly whether the 9/3 element is a suitable element for the analysis; give your reason.

\[
p(x_1, x_2) = p_0 + p_1 x_1 + p_2 x_2
\]