Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12 Introduction to Robotics

Problem Set No.3

Out: October 3, 2005   Due: October 12, 2005

Problem 1

Shown below is a construction robot having two revolute joints and one prismatic joint. Notice that the axis of the prismatic joint has an offset of ℓ_1 from the first revolute joint at the origin O. Namely, the distance between point O and point A is ℓ_1, a constant, and the angle between OA and AB is 90 degrees, a constant as well. Joint 2 is a prismatic joint, whose displacement is given by distance *d*, a variable. Using the geometric parameters and joint displacements shown in the figure, answer the following questions.

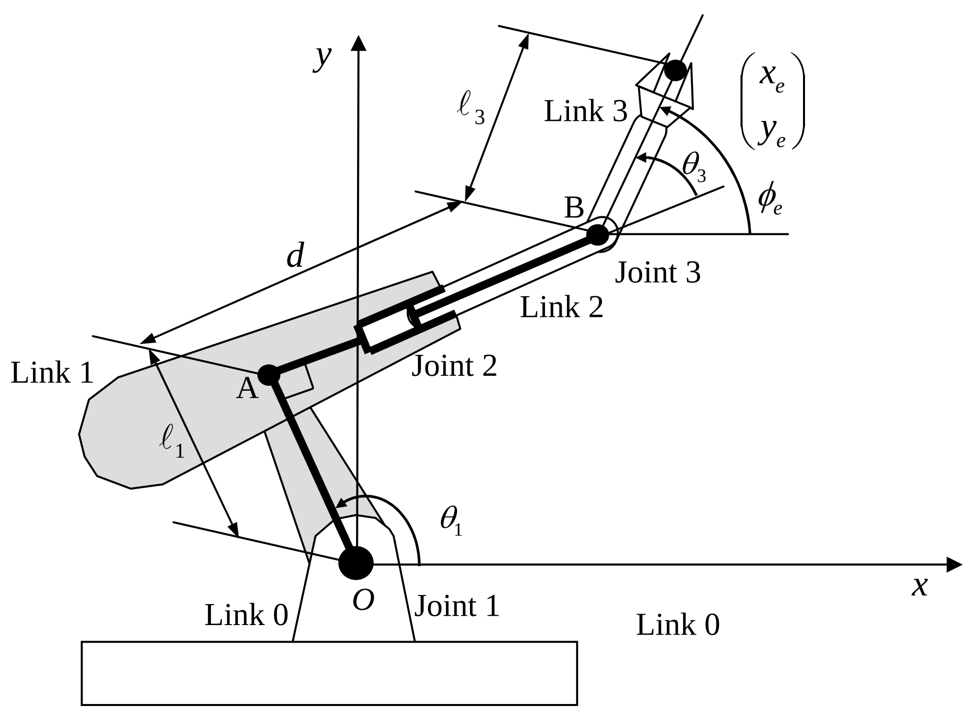

Figure 1 Construction robot with two revolute joints and one prismatic joint

(a) Obtain the kinematic equations relating the end-effecter position and orientation to the joint displacements.

(b) Joint 1 can rotate between 45 degrees and 135 degrees, and joint 3 can rotate from –90 degrees to +90 degrees, while joint 2 can move from ℓ_1 to $2\ell_1$. Sketch the workspace of the end-effecter E within the xy plane.

(c) Solve the inverse kinematics problem to find joint displacements leading the end-effecter to a desired position and orientation: x_e, y_e, ϕ_e.

Problem 2

Shown below is the schematic of a three dof articulated robot arm. Although this arm looks three-dimensional, its kinematic equations can be obtained in the same way as that of planar robots. For joints 2 and 3 alone, consider a vertical plane containing links 2 and 3. As for joint 1, consider the projection of the endpoint onto the *xy* plane. Answer the following questions, using the notation shown in the figure.

(a) Obtain the kinematic equations relating the endpoint coordinates, x_e, y_e, z_e, to joint angles $\theta_1, \theta_2, \theta_3$.

(b) Solve the inverse kinematics problem, i.e. obtain the joint coordinates, given the endpoint coordinates. Obtain all of the multiple solutions, assuming that each joint is allowed to rotate 360 degrees.

(c) Sketch the arm configuration for each of the multiple solutions.

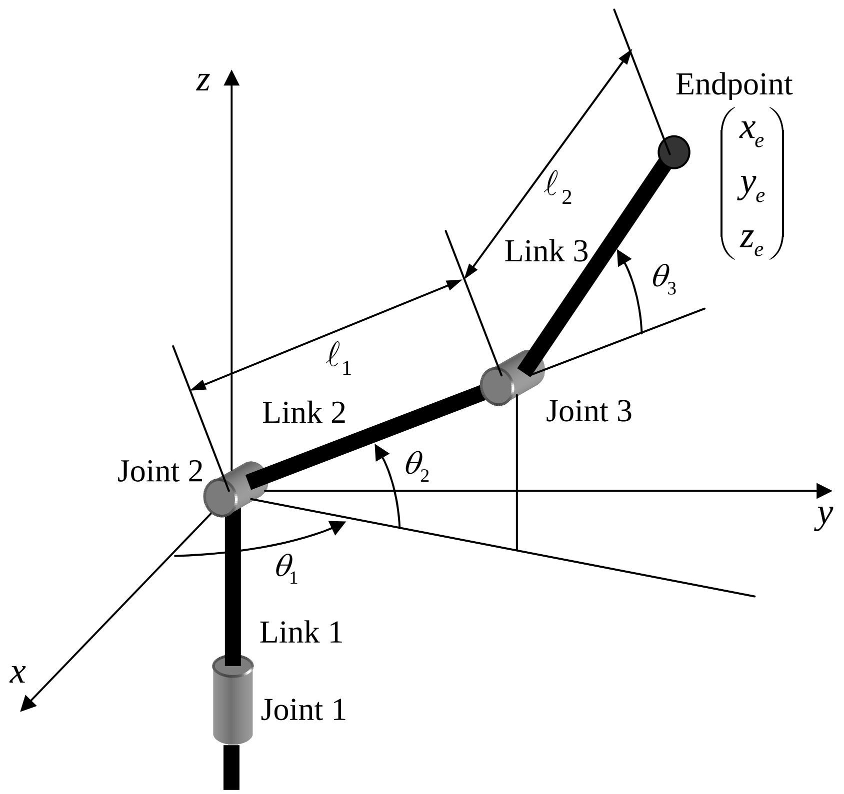

Figure 2 Schematic of 3 dof articulated robot arm

Problem 3

Shown below is a robot arm with three revolute joints. Coordinate system $O-x_0y_0z_0$ is fixed to *Link 0*. Axis x_1 is fixed to *Link 1*. Joint angle θ_1 is measured about the joint axis OA (z_0 axis) from x_0 to x_1. The second joint axis BC is horizontal, and joint angle θ_2 is measured from axis x_1 to axis x_2, which is fixed to *Link 2*, as shown in the figure. Joint angle θ_3 is measured about the joint axis CD from axis x_2 to *Link 3*, i.e. line DE. Link dimensions are OA=1, AB=1, BC=1, CD=0, and DE=1. (For the purpose of explaining the kinematic structure, points C and D are shown to be different points, but they are the same, i.e. the length CD is zero.) Note also that $\angle OAB = \angle ABC = \angle BCD = \angle CDE = 90^o$. Answer the following questions.

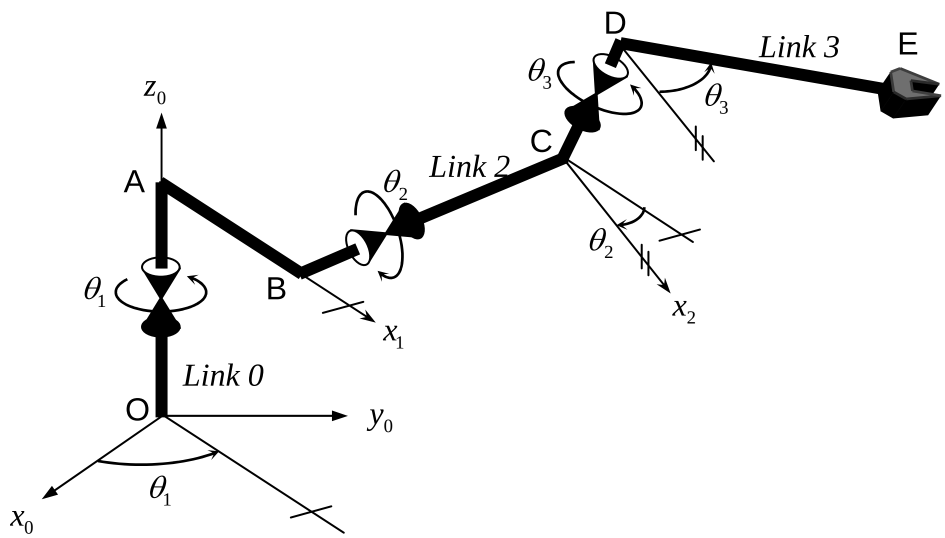

Figure 3 Kinematic structure of 3 DOF robot

a). Obtain the coordinates of point C viewed from the base coordinate system $O-x_0y_0z_0$.

b). Assuming that all the joints are allowed to rotate 360 degrees, determine the workspace of the robot. Sketch the workspace envelope, and show the size and dimensions of the envelope in your sketch.