1. Consider a weather vane in a wind of velocity $U_o$. If $\theta$ is the angle of the vane with respect to the wind direction,

(a) Write the single-degree of freedom ($N$) linearized equations of motion about the fixed axis 0.

(b) Write $N_\theta$, $N_{\dot{\theta}}$, and $N_{\ddot{\theta}}$ in terms of $N_v, N_r, N_r$, etc.

(c) If we consider the differential equation

$$A\ddot{\theta}(t) + B\dot{\theta}(t) + Cy(t) = 0,$$

the condition for stability is that $A$, $B$, and $C$ must have the same sign. Express this requirement in terms of the derivatives in the previous question. Give physical interpretations for what would make such a device stable or unstable.

(d) Create a numerical model of this system, using the MATLAB ODE solver ode45. The system equation can be written as two first-order equations:

$$\frac{d}{dt} \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix} = \begin{bmatrix} -B/A & -C/A \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \end{bmatrix}.$$

Simulate the system response to nonzero initial conditions (e.g., $\theta(0) = 1, \dot{\theta}(0) = 0$). Discuss, using several examples, response sensitivity to $B$ and $C$, which are related to the aerodynamic coefficients. For example, look at the range \{\(A, B, C\)\} = \{1, \pm3, \pm3\}. 

\[\]
2. The figure below shows some characteristic fluid force curves versus a motion parameter. Give the linear hydrodynamic coefficient at two different operating conditions, origin $O$ and $A$: is it zero, small, finite positive, finite negative?