This quiz is open-book. There are four problems, and each is worth 25 points.

Problem 1 (25 points): This problem considers six transfer functions. These are

\[ H_1(z) = \frac{20(z - 0.95)}{z - 1} \]  
\[ H_2(z) = \frac{z - 1}{z - 0.9} \]  
\[ H_3(z) = \frac{z - 0.9}{10(z - 0.99)} \]  
\[ H_4(z) = \frac{10(z - 0.99)}{z - 0.9} \]  
\[ H_5(z) = \frac{1 - 2r_1 \cos \Omega_1 + r_1^2}{z^2 - z2r_1 \cos \Omega_1 + r_1^2} \]  
\[ H_6(z) = \frac{(1 - 2r_1 \cos \Omega_1 + r_1^2)(z^2 - z2r_2 \cos \Omega_2 + r_2^2)}{(z^2 - z2r_1 \cos \Omega_1 + r_1^2)(1 - 2r_2 \cos \Omega_2 + r_2^2)} \]

where \( r_1 = 0.995 \) and \( \Omega_1 = 0.1 \).

On two pages attached to the end of this exam are six step responses and six frequency response (Bode) plots. These plots are labeled I, II, III, IV, V, VI; and A, B, C, D, E, F; respectively. For each of the transfer functions above, indicate which are the corresponding step and frequency responses. Your answer should take the form of a number from 1-6 for each transfer function followed by a capital letter indicating the corresponding step response, followed by a Roman numeral indicating the corresponding frequency response. Wrong answers will count as zero; no partial credit will be given in this problem.
Problem 2 (25 points): Consider a continuous time filter:

\[ H(s) = \frac{10(s + 1)}{(0.5s + 1)(0.05s + 1)} \]

a) Use the forward Euler approximation to map this to an approximating discrete-time system for \( T = 0.1 \text{sec} \).

b) What is the approximating difference equation?

c) Plot the poles in the z-plane. Comment on the quality of the approximation.
Problem 3 (25 points): A given linear, time-invariant (LTI) system H has an impulse response s(k) which is shown below:

Note that the only non-zero values of s(k) are at k = 1, 3, 4, 5.

a) Use any convenient method to determine the difference equation for this system in terms of an input e(k) and an output y(k). Please explain your approach.

b) Draw a minimum block diagram realization of the system from the input to the output.

c) Now suppose that the system input is a unit step, i.e., e(k) = u_s(k). Using whatever approach you find most efficient, write an expression for the resulting output y(k) which is valid for all k.

d) Find the z-transform S(z) of s(k) as given in the figure above. How is this transform related to the the system transfer function H(z)?
**Problem 4 (25 points):** A block diagram for a system with a varying plant model and a fixed lead compensator is shown below:

![Block Diagram](image)

\[ G_c(z) = K \frac{z^{-0.5}}{z^{-0.2}} \]

Make reasonably accurate root locus plots for \( K \geq 0 \).

a) \( G_p(z) = \frac{1}{(z-1)^2} \)

b) \( G_p(z) = \frac{1}{(z^2 - 2R_0 \cos(\Omega_0)z + R_0^2)(z-1)} \) where \( \Omega_0 = \frac{\pi}{4}, R_0 = 0.9 \)

c) \( G_p(z) = \frac{z^2 - 2R_1 \cos(\Omega_1)z + R_1^2}{(z^2 - 2R_2 \cos(\Omega_2)z + R_2^2)(z-1)} \) where \( R_1 = 0.99, \ Omega_1 = \frac{\pi}{3}, \ Omega_2 = \frac{32}{100} \pi, \ R_2 = R_1 \frac{\cos(\Omega_1)}{\cos(\Omega_2)} \)