Homework 5
2.18/2.180
Due Wednesday March 18 at beginning of class

Problems with a star (*) are for graduate students only.

Problem 1: (5 points) Exercise 3.3

Problem 2: (5 points) Consider the two abstract “motifs” of incoherent feedforward loops given in Section 3.2. Propose one implementation for each of the two motifs, using the core processes seen in Chapter 2 to derive the corresponding ODE models, and demonstrate that the implementations work. That is, show that the output $y$ asymptotically reaches a constant value $y_0$ that is independent of the disturbance input $u$. Make sure you clarify the assumptions that are important for the implementation to work.

Problem 3(*): Consider the following model of a nonlinear harmonic oscillator (Fig 3.15a):

\[
\begin{align*}
\dot{x}_1 &= x_2 + x_1(1 - x_1^2 - x_2^2) \\
\dot{x}_2 &= -x_1 + x_2(1 - x_1^2 - x_2^2)
\end{align*}
\]

Answer the following questions:

(i) demonstrate that the system admits a limit cycle;

(ii) demonstrate that this limit cycle is asymptotically stable

(Hint: using radial coordinates may be useful).

Consider now the following modification to the above model:

\[
\begin{align*}
\dot{x}_1 &= x_2^3 \\
\dot{x}_2 &= -x_1^3
\end{align*}
\]

(iii) Demonstrate that this system also admits a limit cycle

(iv) Determine whether the limit cycle is stable, unstable, or asymptotically stable.
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